Exam
Set theory, LOG120

2018–11–01

This exam is marked and graded anonymously using code numbers. Please enter your name and personal identity number below. Then enter only the code number on each answer sheet.

Name / Namn: . . . .

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FLoV
Göteborgs Universitet

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No aids are permitted.

1. Give, if possible, examples of sets, whose existence is provable in ZF such that (3p)
   (a) $x \in y$ and $x \nsubseteq y$
   (b) $x \subseteq y$ and $x \notin y$
   (c) $x \in y$ and $x \subseteq y$.
2. Let R be a relation on $\mathbb{N} \times \mathbb{N}$, i.e., $R \subseteq (\mathbb{N} \times \mathbb{N}) \times (\mathbb{N} \times \mathbb{N})$, such that $(x, y)R(z, w)$ iff $x \leq z$ and $y \leq w$. (3p)
   (a) Is R a total order (linear order)?
   (b) Is there a maximal and/or least element?
   (c) Does every subset of $\mathbb{N} \times \mathbb{N}$ has a least element?
3. (a) Give the definition of two sets X and Y having the same cardinality, i.e., of $X \approx Y$. (4p)
   (b) Find a proper subset $A \subsetneq \mathbb{N}$ such that $A \approx \mathbb{N}$. Prove it.
4. (a) Let A and B be sets, what is A^B? (3p)
   (b) Give an explicit description of $A^{\emptyset}$ by listing all its elements.
5. (a) What is an ordinal nmumber? Define. (3p)
   (b) What is a transitive set? Define.
   (c) Give an example of a transitive set that is not an ordinal number.
6. (a) Define the notion $A \preceq B$ and $A \prec B$? (4p)
   (b) Prove directly, *without* using Cantor's theorem, that $\mathbb{N} \prec 2^{\mathbb{N}}$.
7. (a) If A and B are well-ordered by $<_A$ and $<_B$ respectively, defined $A+B$ and $A \times B$. (4p)
   (b) Which well-orders A are such that $1 + A \cong A$?
   (c) Which well-orders B are such that $2 \times B \cong B$?

Max points: 24. 12 points are required for Pass (G) and 18 for Pass with distinction (VG).

Fredrik Engström, `fredrik.engstrom@gu.se`, 031 – 786 6335