Exam
Logical theory part I, LOG110

2018−10−30

This exam is marked and graded anonymously using code numbers. Please enter your name and personal identity number below. Then enter only the code number on the answer sheets.

Name / Namn: . . . .

Personal identity number / Personnummer: . . . .

Code number / Tentamensnummer: . . . .

No aids are permitted.

1. Derive the following sentences using natural deduction: (4p)

   (a) $\neg\neg p \to p$

   (b) $(p \vee \bot) \to p$

   (c) $\forall x(\varphi(x) \vee \psi) \to (\forall x\varphi(x) \vee \psi)$, where x does not occur in ψ.

2. Show that   (4p)

   (a) $\nvdash \forall x \exists y P(x, y) \to \exists y \forall x P(x, y)$, and

   (b) $\nvdash (\forall x Q(x) \to R) \to \forall x(Q(x) \to R)$.

3. Prove that $\varphi, \psi \models \sigma$ iff $\models (\varphi \wedge \psi) \to \sigma$.   (3p)

4. Prove that the following statements are equivalent, e.g., by proving that   (4p)
   (a) implies (b), (b) implies (c) and (c) implies (a):

   (a) Γ is consistent, i.e., $\Gamma \nvdash \bot$.

   (b) There is no φ such that $\Gamma \vdash \varphi$ and $\Gamma \vdash \neg\varphi$.

   (c) There is a sentence φ such that $\Gamma \nvdash \varphi$.

5. Consider $\exists x P(f(x)) \to (\neg\forall y P(y) \to \exists x f(x) = x)$ and find   (3p)

   (a) a prenex normal form, and

   (b) a Skolem form.

6. Let Γ be a finite set of sentences. Describe how to construct a maximally   (3p)
   consistent extension T of Γ and prove that the extension is complete, i.e.,
   that for each sentence φ in the language of T: $T \vdash \varphi$ or $T \vdash \neg\varphi$.

7. Let $\underline{A} = \langle A, \sim \rangle$, where $A = \mathbb{N} \times \mathbb{N}$ and $\langle n, k \rangle \sim \langle n', k' \rangle$ iff $k = k'$ and let   (3p)
   $T = \text{Th}(\underline{A})$. Show that if $\underline{B} \models T$ is a countable model of T then $\underline{A} \cong \underline{B}$
   and write down a set of sentences Γ such that $\Gamma \equiv T$.

Max points: 24. 12 points are required for Pass (G) and 18 for Pass with distinction (VG).

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