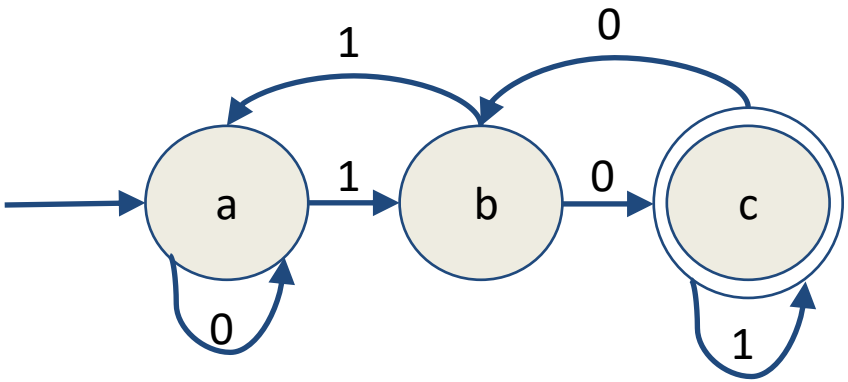
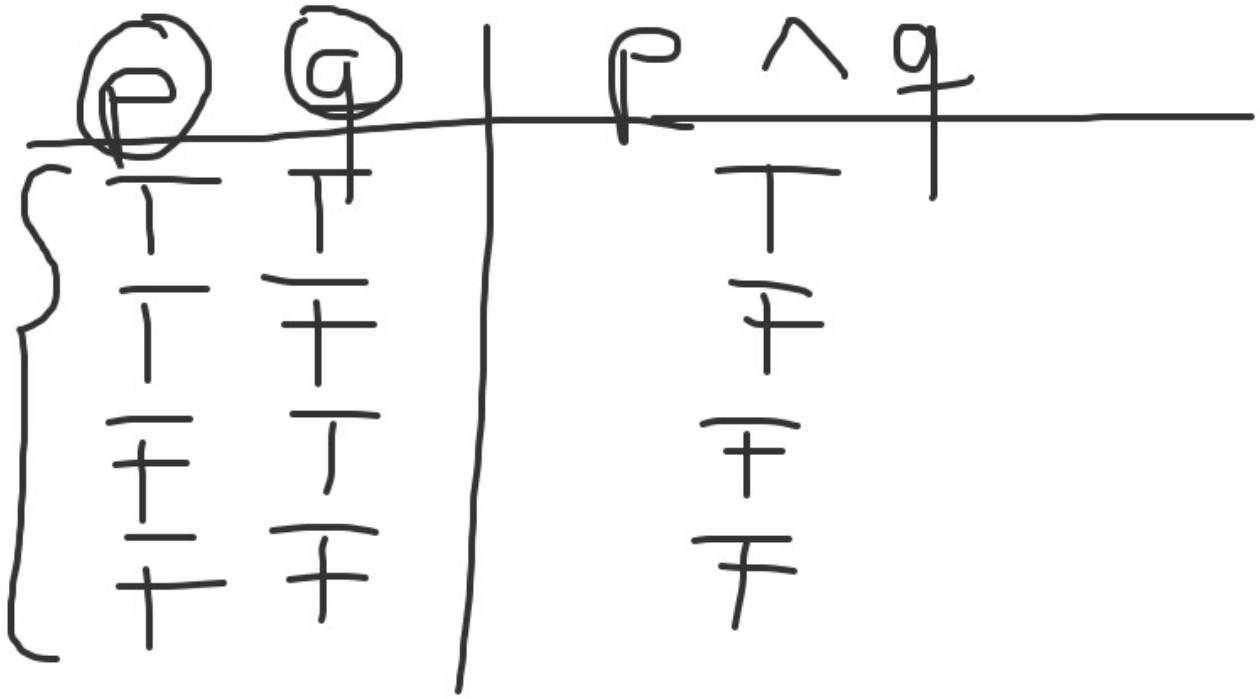



DIT-022

Mathematical Foundations
for Software Engineering

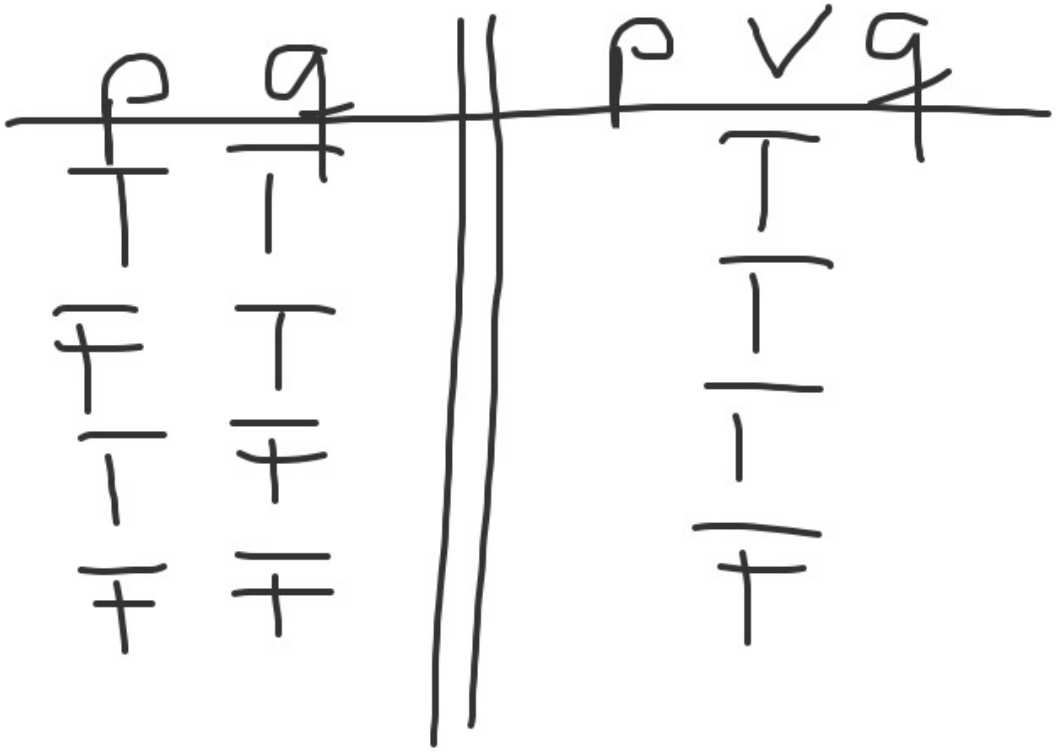
Logic

Def 2: conjunction

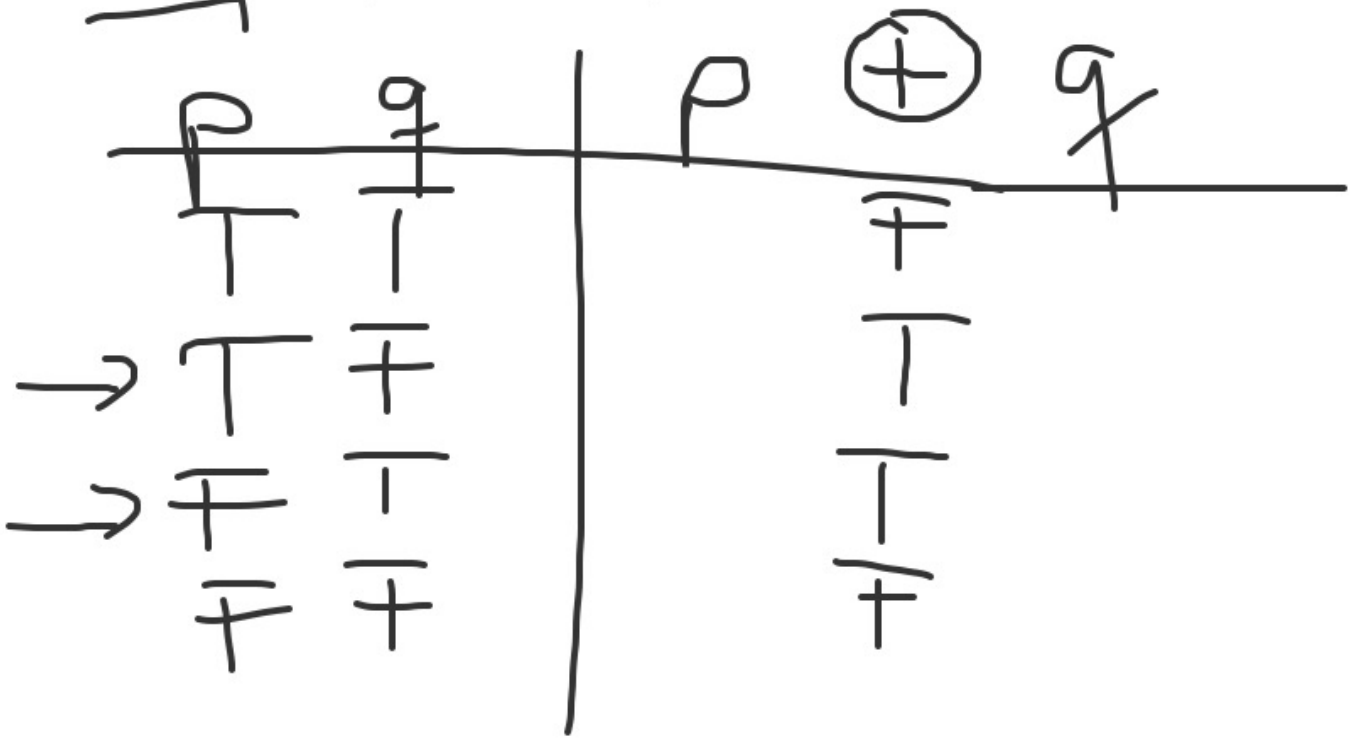


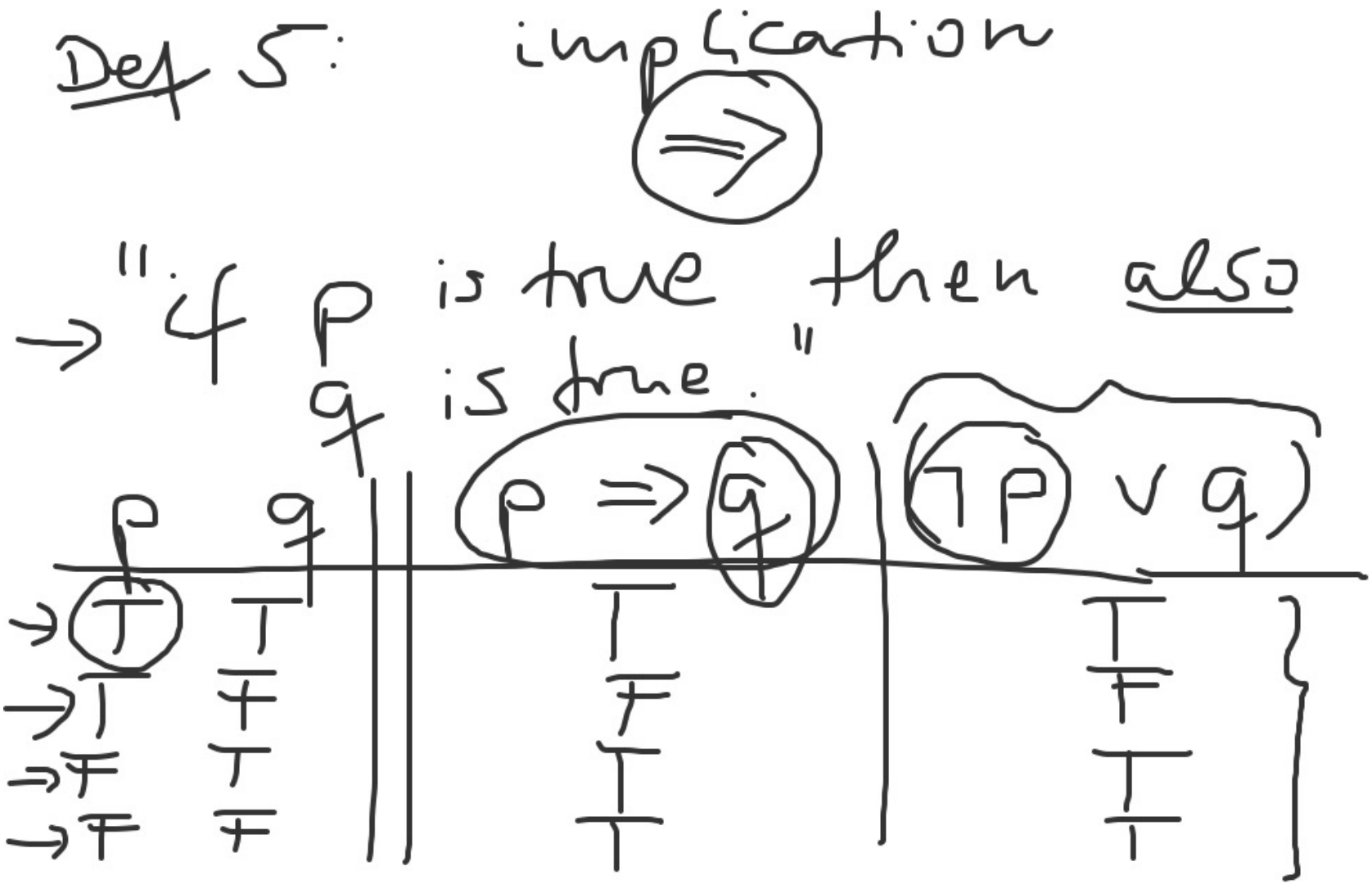

p, q, r  3  $z^3$  8

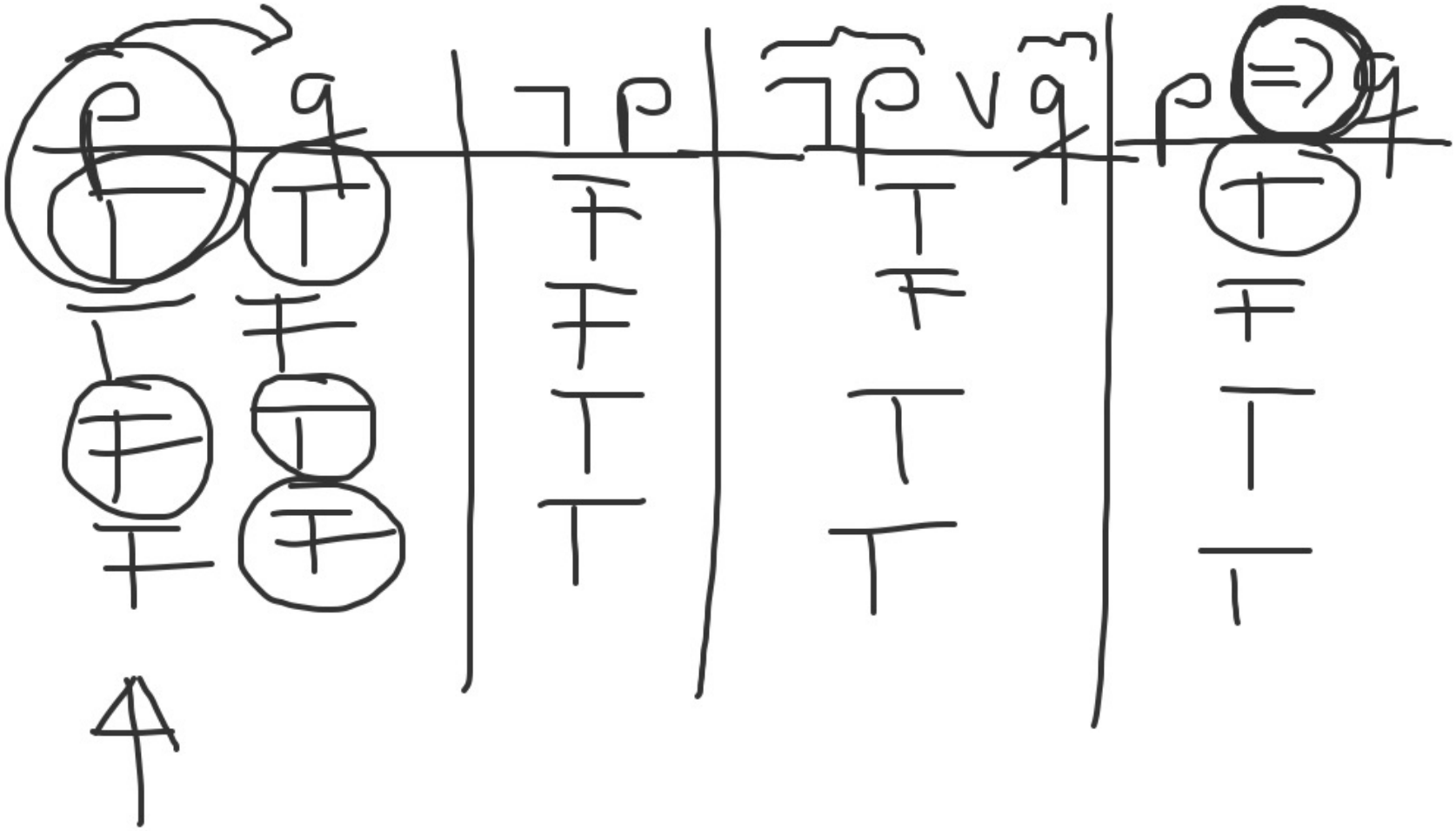
Def 3: Disjunction



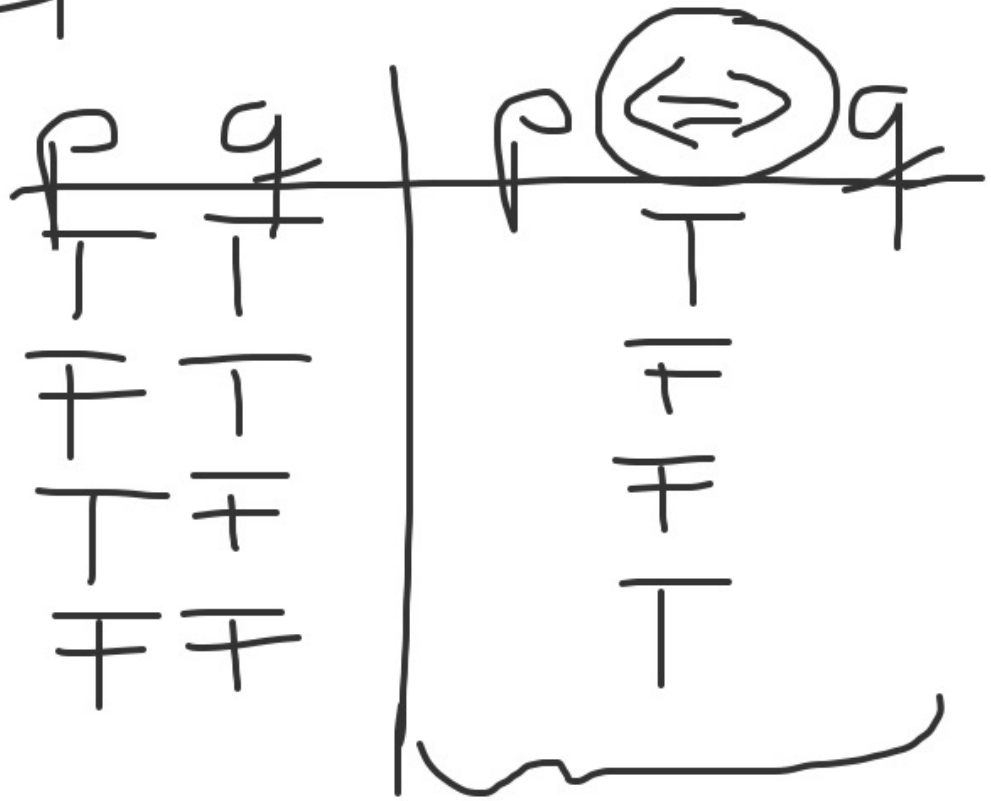
Def 4: XOR




Def 5: implication
⇒
→ "if p is true then also q is true."
p
q
p ⇒ q
¬p ∨ q
T
T
T
T
T
F
F
T
F
F
F
T

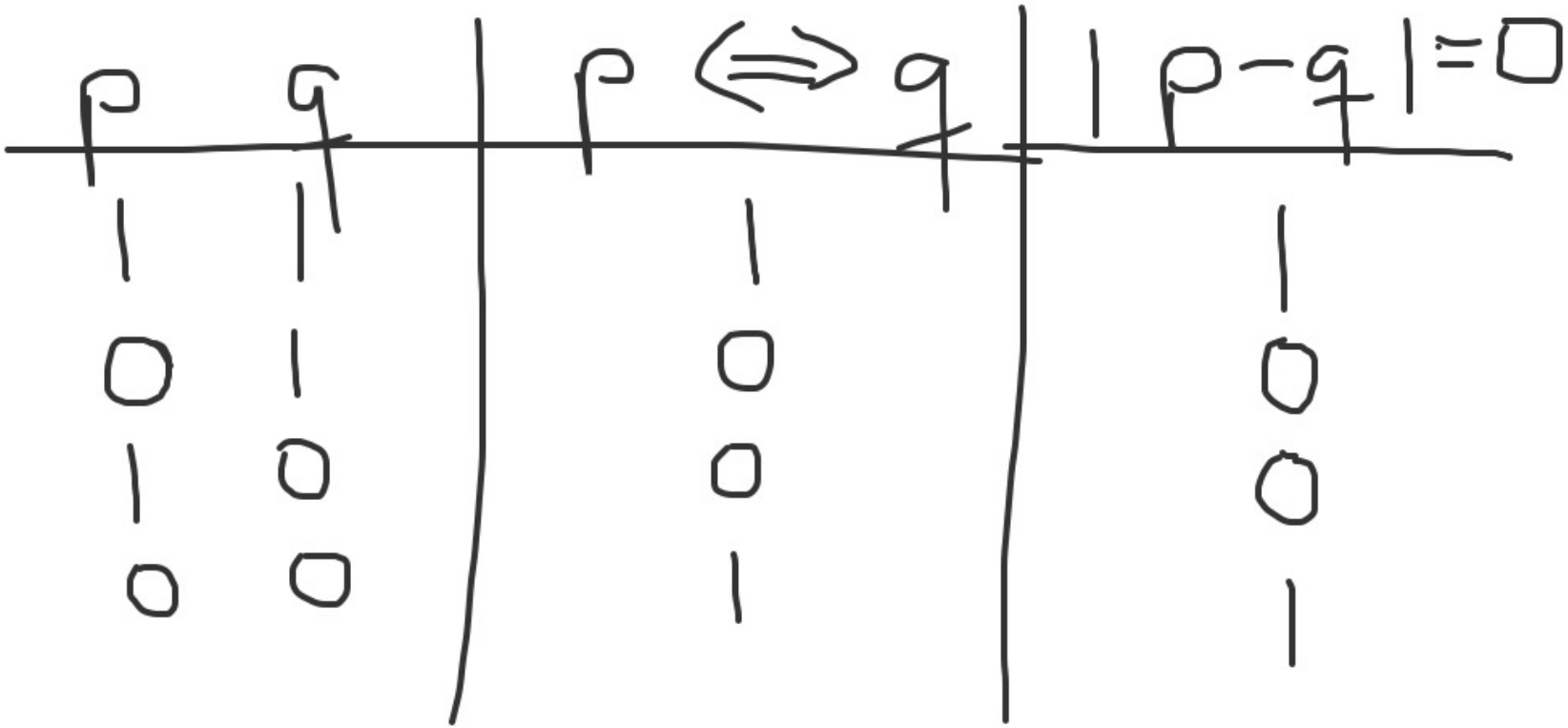

q
¬p
¬p ∨ q
p ⇒ q

Def 6: b: conditional statement




"iff"

if and only if

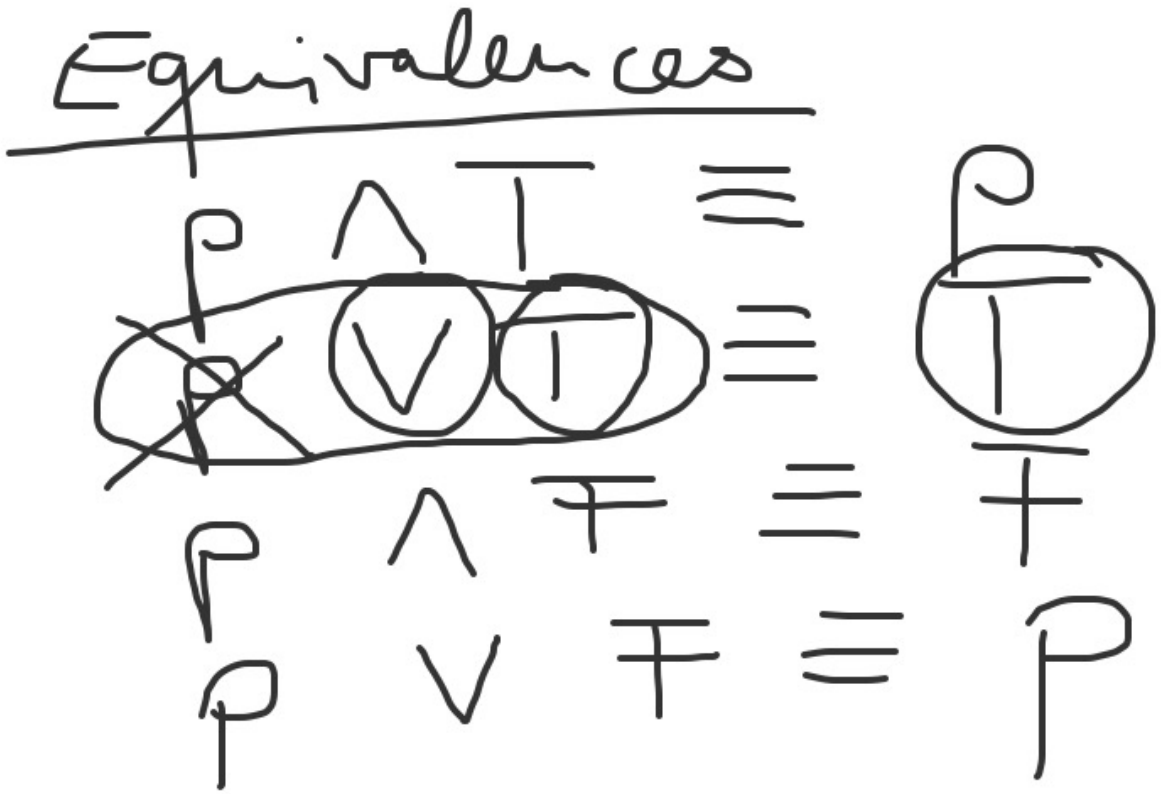

p
q
p ⟺ q
|p-q|=0
1
0
1
0
1
0
0
1
1
0
0
1

tautology: always true

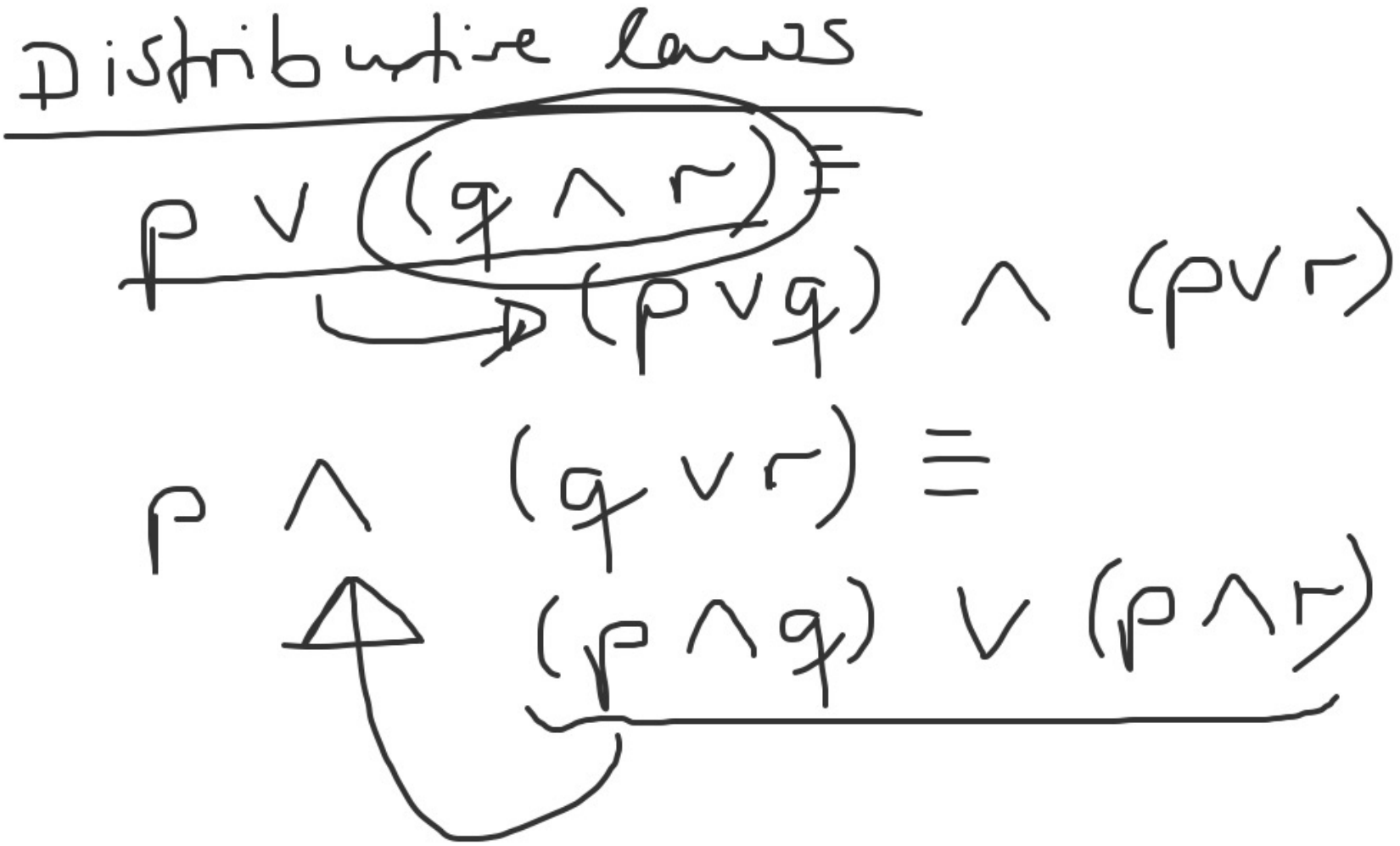
contradiction: always false

if c.p. is not a contradiction

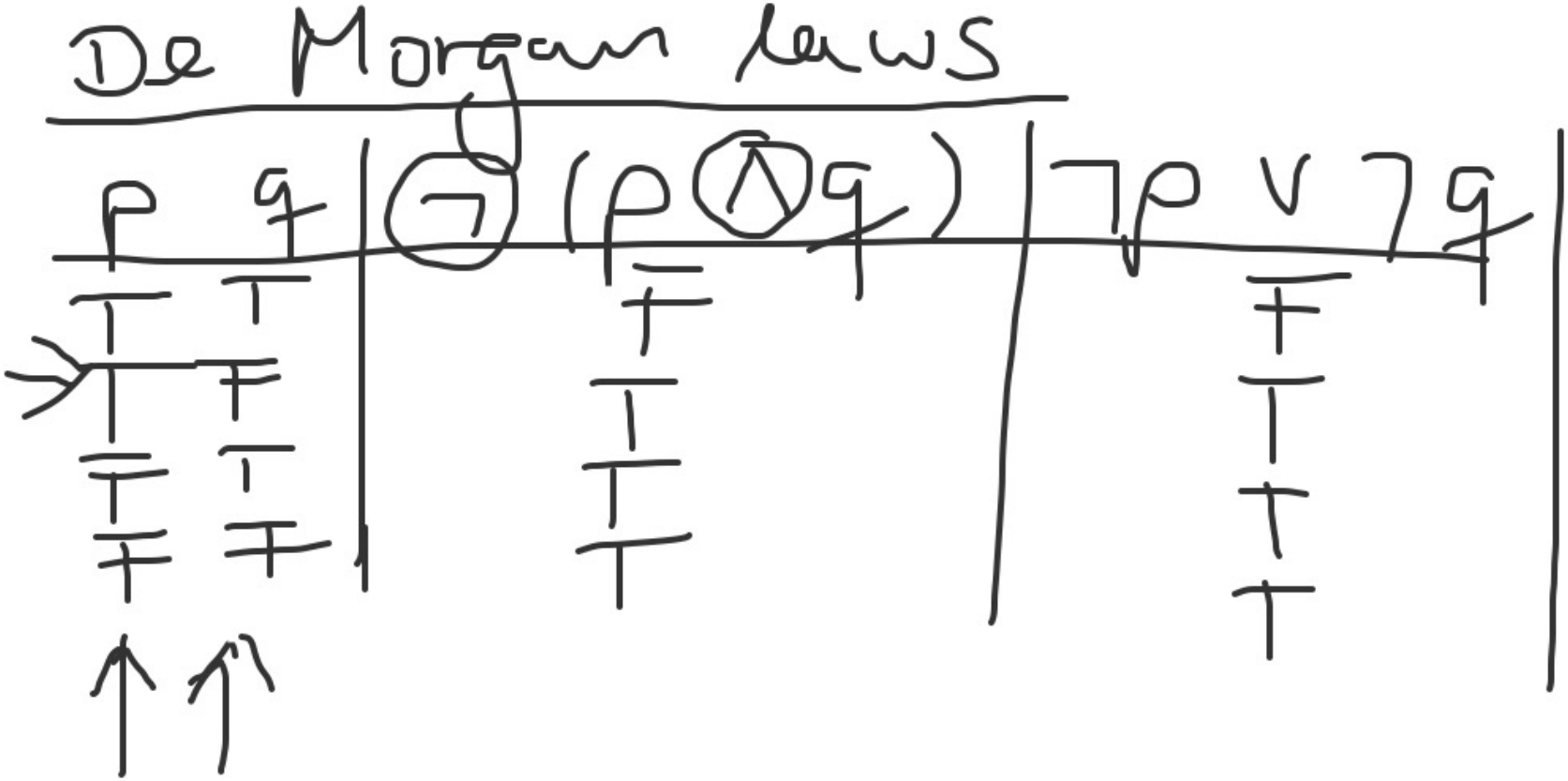
$\Rightarrow$ tautology? NO


Equivalences

# Distributive laws



# De Morgan laws



satisfiable $p, q, r, \ldots$

$\top$

not satisfiable

not tautology

$\top \equiv \{ (p^{\downarrow} \wedge q^{\downarrow}) \vee r^{\downarrow} \Rightarrow (\ldots) \}$

**Def**

connectives: $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$

constants: $a, b, c$

variables: $x, y, z$

predicate symbol: $P, Q, R$

functions: $f, g$

quantifiers: $\forall, \exists$

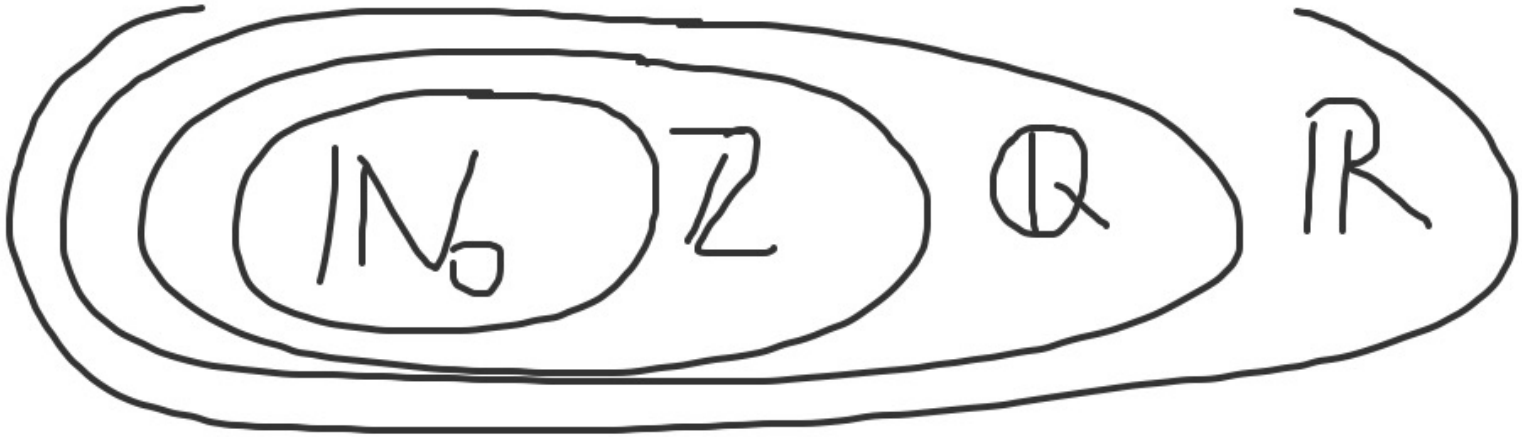
identity: $=$

## "universe of discourse"

$$\mathbb{N}_0 = \{0, 1, 2, 3, \ldots\}$$

$$\mathbb{N}_1 = \{1, 2, 3, \ldots\}$$

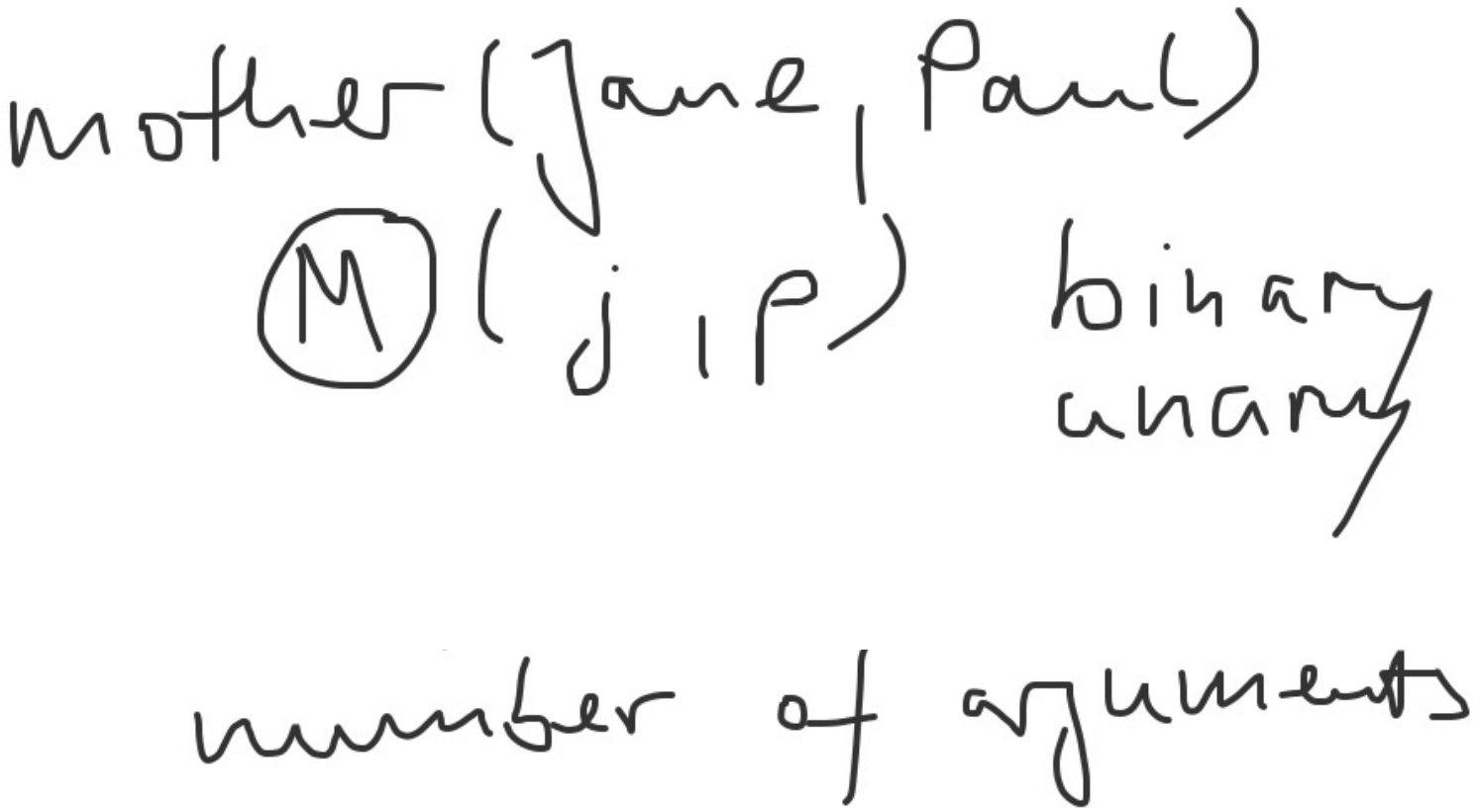
$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, \ldots\}$$

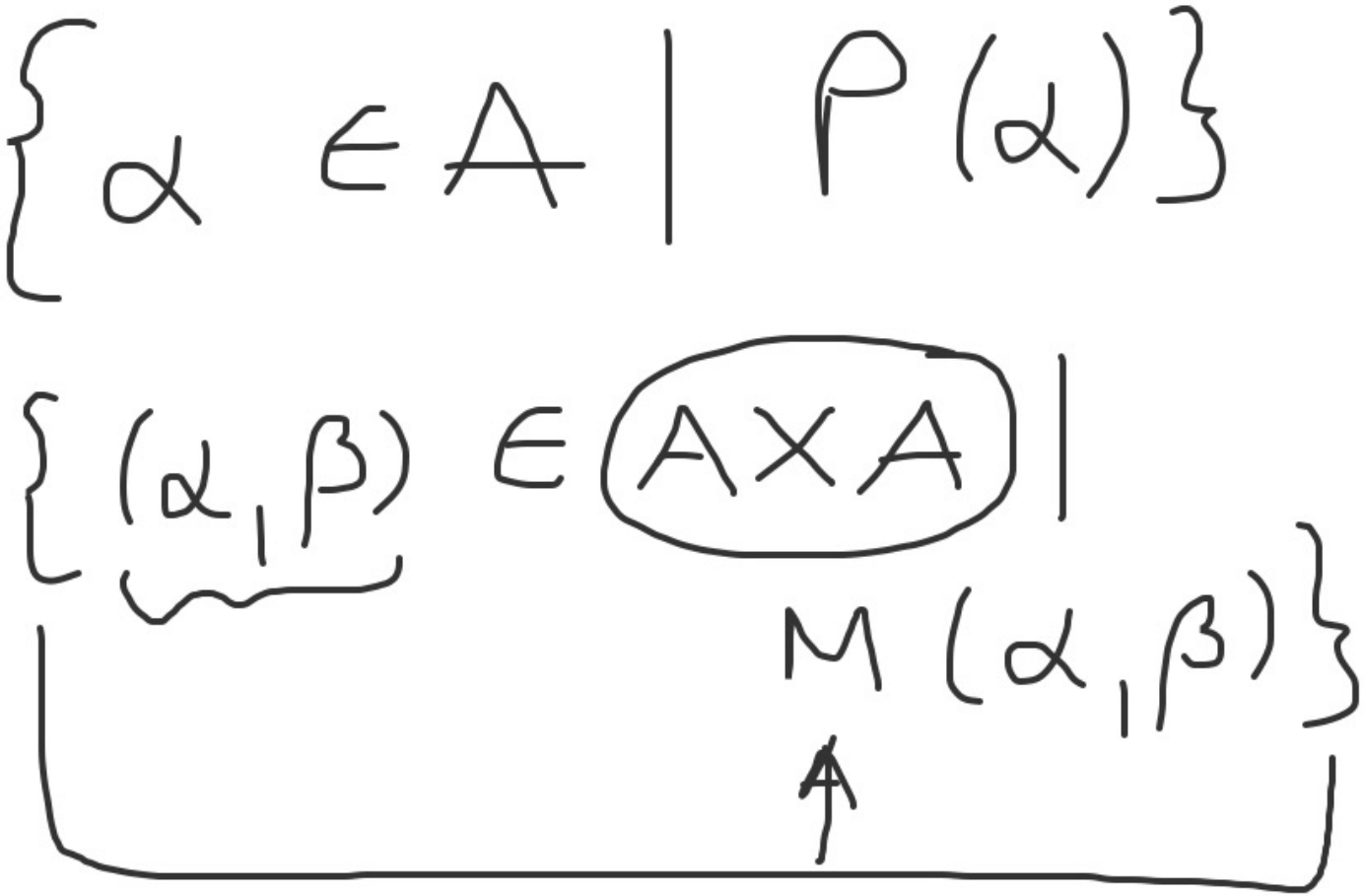


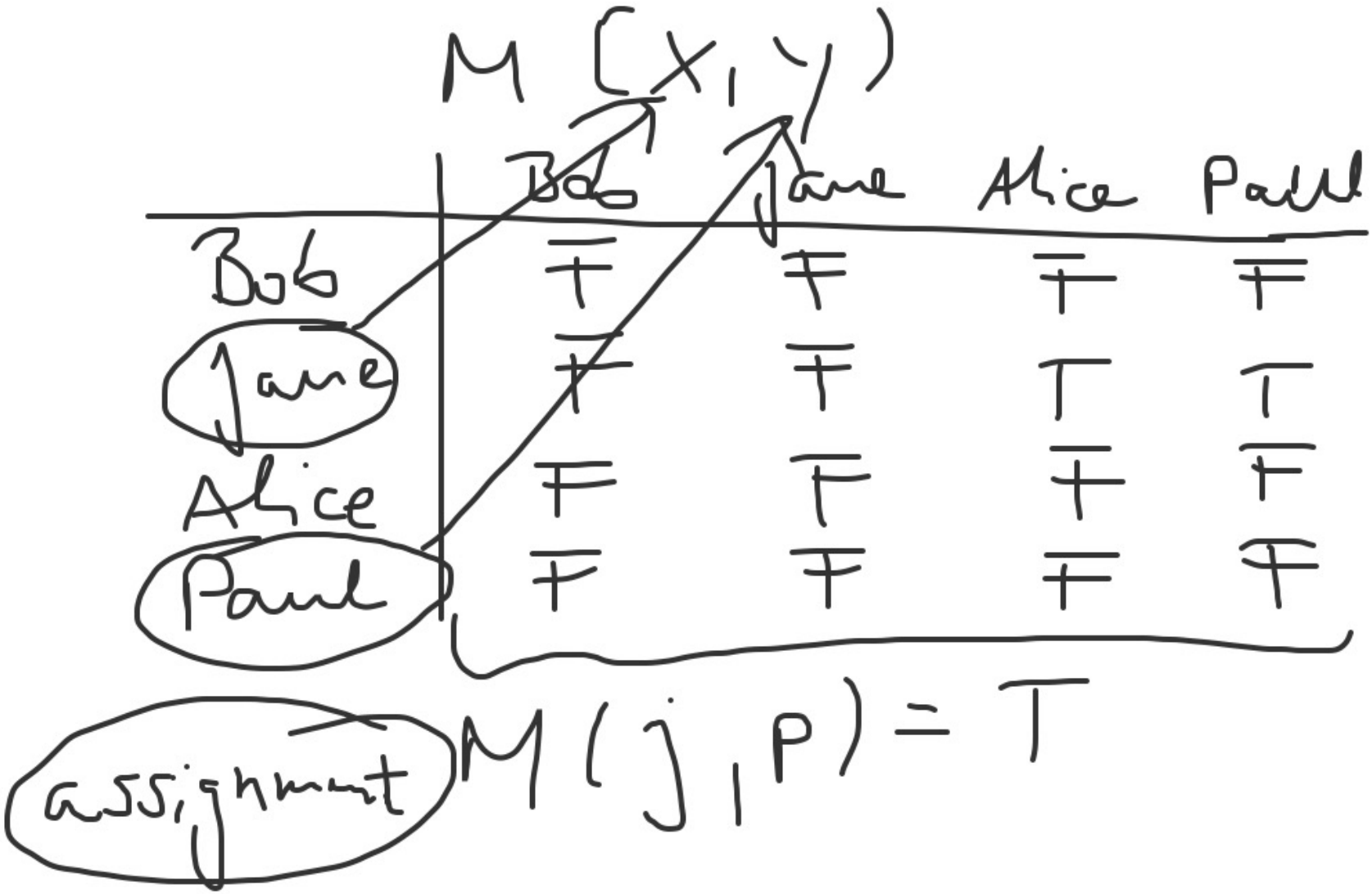

$\mathbb{Q}$ $\frac{p}{q}$ $\frac{3}{4}$ $\frac{2}{3}$

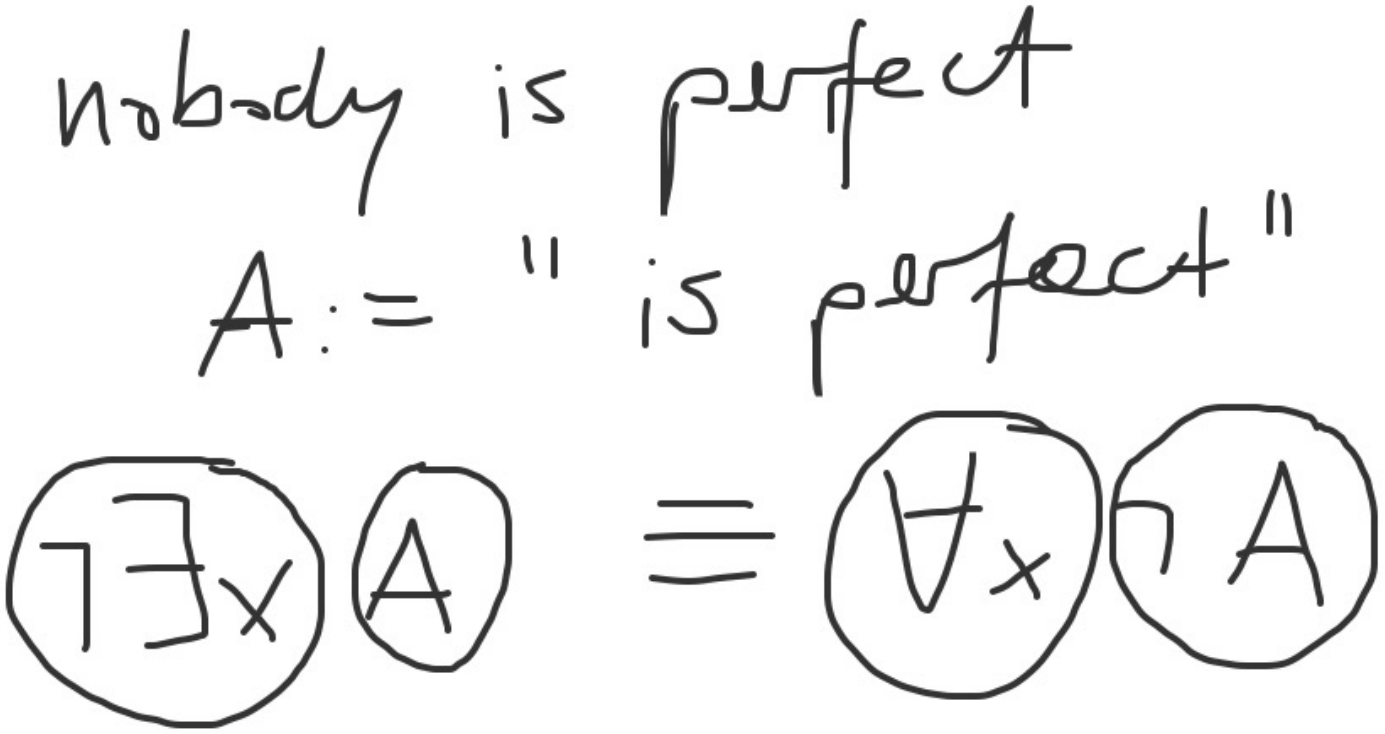
$= 1$

$\neq 0$


mother(Jane, Paul)
M(j, p)
binary
unary
number of arguments


{α ∈ A | P(α)}
{(α, β) ∈ A × A |
M(α, β)}


M(x, y)
Bob
Jane
Alice
Paul
Bob
Jane
Alice
Paul
assignment
M(j, p) = T


nobody is perfect
A := " is perfect "
¬∃x A ≡ ∀x ¬A

$\forall$ : sth is true for all individuals

$\exists$ : sth is true for some individual

$B(x) :=$ "gets a break once in a while"

$\forall x\, B(x)$

all cats have tails

if cat(x) then hasTail(x)

$$\forall x \left( cat(x) \Rightarrow hasTail(x) \right)$$

$\exists x A$

$P(x) := \text{"eats meat"}$

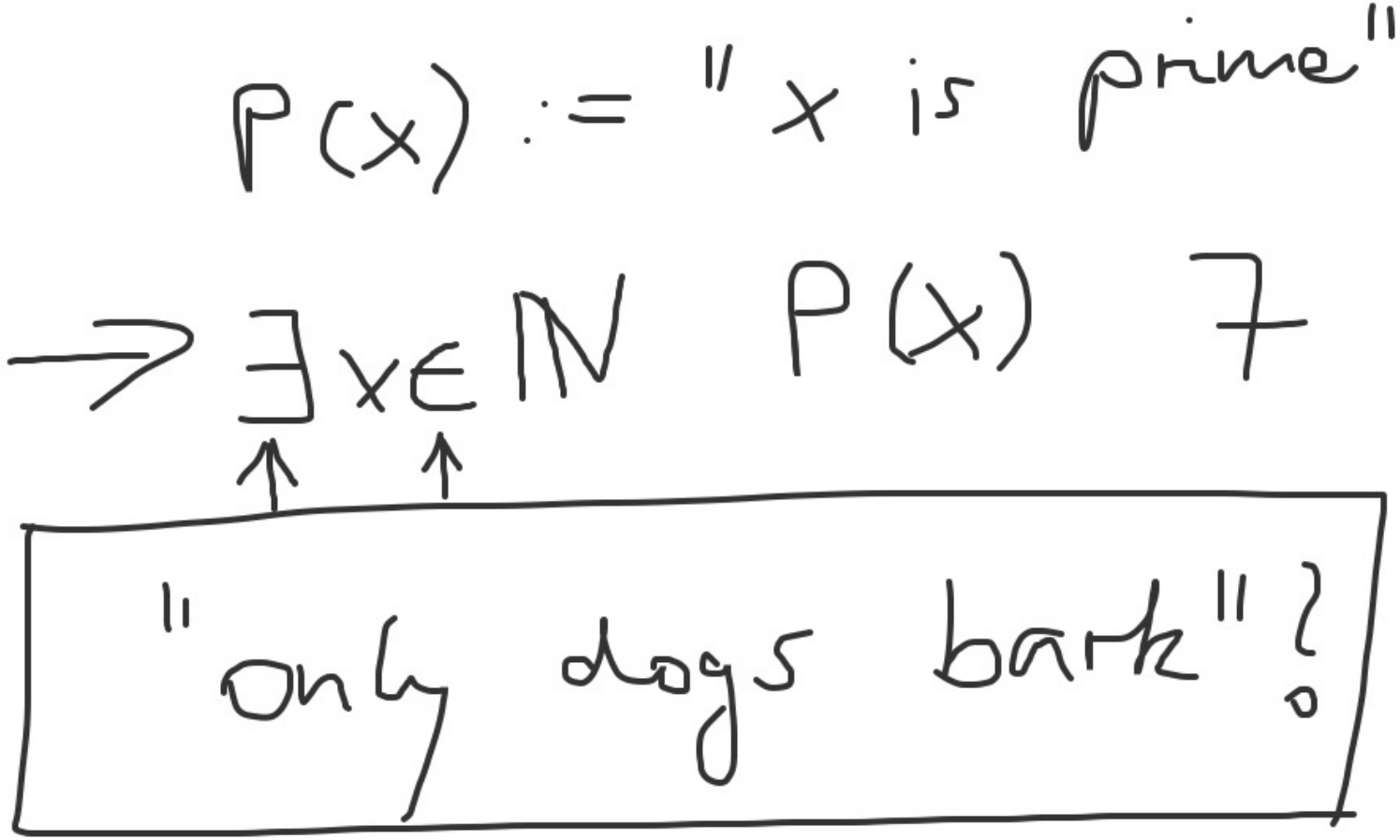
$\exists x (\neg P(x))$

$P(x) :=$ "$x$ is living"

$Q(x) :=$ "$x$ is dead"

$$\forall x \, (P(x) \vee Q(x))$$

$$\forall x \, P(x) \vee Q(x)$$


P(x) := "x is prime"
⇒ ∃ x ∈ ℕ P(x) ?
"only dogs bark"!