

DIT-022

Mathematical Foundations  
for Software Engineering

Logic

Def 2: conjunction

$P$	$Q$	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

$P, Q, \neg$  3  $2^3$  8

# Def 3: Disjunction

p	q		p	$\vee$	q
T	T		T	T	T
T	F		T	T	F
F	T		F	T	T
F	F		F	F	F

Def 4: XOR

	p	q		p	$\oplus$	q
	1	1		1	1	1
→	1	1		1	1	1
→	1	1		1	1	1
	1	1		1	1	1

Def 5:

implication



→ "if  $p$  is true then also  
 $q$  is true."

$p$	$q$	$p \Rightarrow q$	$(\neg p) \vee q$
→ T	T	T	T
→ T	F	F	F
→ F	T	T	T
→ F	F	T	T



Def 6: b:conditional statement

p	q	p $\Leftrightarrow$ q
T	T	T
T	F	F
F	T	F
F	F	T

"iff"  
if and  
only if

p	q	$p \Leftrightarrow q$		$ p - q  = 0$	
1	1	1		1	
0	1	0		0	
1	0	0		0	
0	0	1		1	



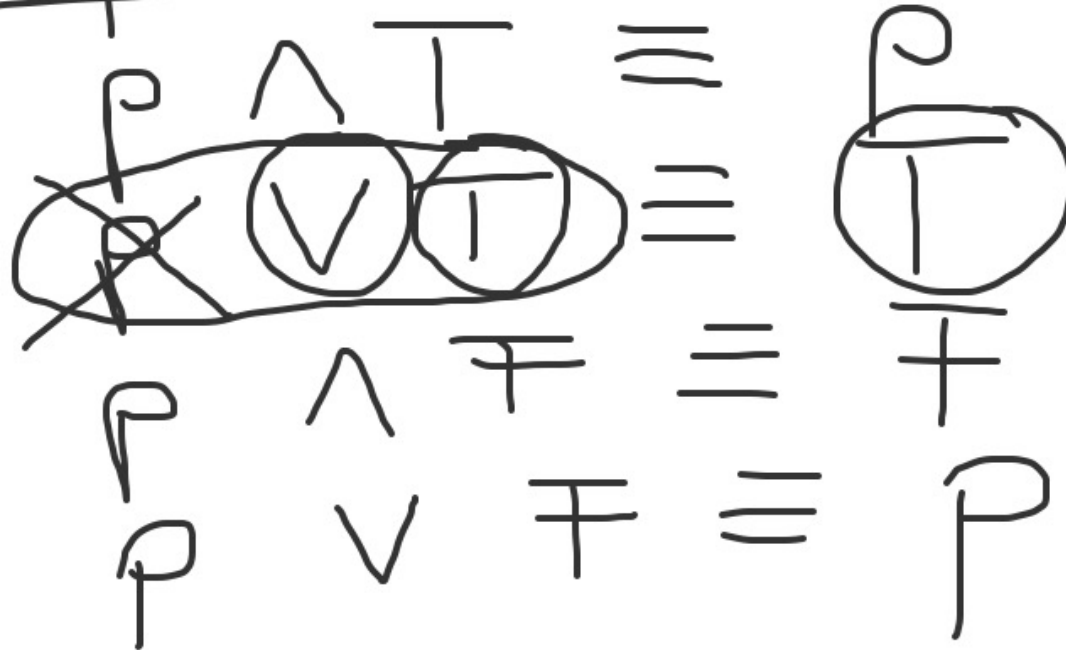
tautology: always true

contradiction: always  
false

if c.p. is not  
a contradiction

$\Rightarrow$  tautology? NO

# Equivalences



## Distributive laws

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

# De Morgan laws

$P$	$q$	$\neg(P \wedge q)$	$\neg P \vee \neg q$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

$\nearrow$   $\nearrow$

satisfiable

$p, q, r, \dots$

not satisfiable

not tautology

$$T \equiv \{ (\overset{\downarrow}{p} \wedge \overset{\downarrow}{q}) \vee \overset{\downarrow}{r} \Rightarrow (\dots) \}$$

Def connectives:  $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$   
constants:  $a, b, c$   
variables:  $x, y, z$   
predicate symbol:  $P, Q, R$   
functions:  $f, g$   
quantifiers:  $\forall, \exists$   
identity:  $=$

"universe of discourse"

$$\mathbb{N}_0 = \{\underline{0}, 1, 2, 3, \dots\}$$

$$\mathbb{N}_1 = \{1, 2, 3, \dots\}$$

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, \dots\}$$



$Q$

$\frac{p}{q}$

$\frac{3}{4}$

$\frac{2}{3}$

$\neq 0$



mother(jane, Paul)

(M)(j, P)

binary  
unary

number of arguments

$$\{\alpha \in A \mid P(\alpha)\}$$

$$\{\underbrace{(\alpha, \beta)}_{\substack{\uparrow \\ A}} \in \underbrace{A \times A}_{M(\alpha, \beta)} \mid \dots\}$$

$M(x, y)$

	Bob	Jane	Alice	Paul
Bob	F	F	F	F
Jane	F	F	T	T
Alice	F	F	F	F
Paul	F	F	F	F

Arrows from the circled 'Jane' and 'Paul' in the left column point to the 'Jane' and 'Paul' headers in the top row, respectively.

assignment  $M(j, p) = T$

nobody is perfect

$A :=$  "is perfect"

$$\neg \exists x (A) \equiv \forall x (\neg A)$$

$\forall$  : sth is true for all individuals

$\exists$  : sth is true for some individuals

$B(x) :=$

$\forall x B(x)$

"gets a break  
once in a  
while"

all cats have tails  
if  $\text{cat}(x)$  then  
 $\text{hasTail}(x)$

$$\forall x (\text{cat}(x) \Rightarrow \text{hasTail}(x))$$

$$\exists x A$$

$$P(x) := \text{"eats meat"}$$

$$\exists x (\neg P(x))$$

$P(x) := "x \text{ is living}"$   
 $Q(x) := "x \text{ is dead}"$

$\forall x (P(x) \vee Q(x))$

$\forall x P(x) \vee Q(x)$



$P(x) := "x \text{ is prime}"$

$\rightarrow \exists x \in \mathbb{N} \quad P(x) \quad \neg$

"only dogs bark"?