Exercise 1.1.

Suppose that during the most recent fiscal year, the annual revenue of Acme Computer was 138 billion dollars and its net profit was 8 billion dollars, the annual revenue of Nadir Software was 87 billion dollars and its net profit was 5 billion dollars, and the annual revenue of Quixote Media was 111 billion dollars and its net profit was 13 billion dollars. Determine the truth value of each of these propositions for the most recent fiscal year.

a) Quixote Media had the largest annual revenue.
b) Nadir Software had the lowest net profit and Acme Computer had the largest annual revenue.
c) Acme Computer had the largest net profit or Quixote Media had the largest net profit.
d) If Quixote Media had the smallest net profit, then Acme Computer had the largest annual revenue.
e) Nadir Software had the smallest net profit if and only if Acme Computer had the largest annual
revenue.

	Rev	Net
Acme	- 138 billion	8 billion
Nadir	- 87 billion	5 billion
Quixote	- 111 billion	13 billion

Our answers:

a) - F
b) T $\wedge$ T - T
c) F $\vee$ T - T
d) F $\Longrightarrow$ T - T
e) T $\Longleftrightarrow$ T - T

Exercise 1.2.

Let **p** and **q** be the propositions

p: You drive over 65 miles per hour.

q: You get a speeding ticket.

Write these propositions using p and q and logical connectives (including negations).

a) You do not drive over 65 miles per hour.

b) You drive over 65 miles per hour, but you do not get a speeding ticket.

c) You will get a speeding ticket if you drive over 65 miles per hour.

d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.

e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.

f) You get a speeding ticket, but you do not drive over 65 miles per hour.

g) Whenever you get a speeding ticket, you are driving over 65 miles per hour.

Our answers:

a) ¬p

b) p ∧ ¬q

c) p $\Longrightarrow$ q

d) $\neg p \implies \neg q$

e) $p \implies q$

f) $q \wedge \neg p$

g) $q \implies p$

Exercise 1.3. Construct a truth table for each of the following compound propositions.

a) $p \to (\neg q \vee r)$

b) $\neg p \to (q \to r)$

c) $(p \to q) \vee (\neg p \to r)$

d) $(p \to q) \wedge (\neg p \to r)$

e) $(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$

f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

Our answers:

A) B) C) D) E) F)

p	q	r	$p \implies (\neg q \vee r)$	$\neg p \implies (q \implies r)$	$(p \implies q) \vee (\neg p \implies r)$	$(p \implies q) \wedge (\neg p \implies r)$	$(p \iff q) \vee (\neg q \iff r)$	$(\neg p \iff \neg q) \iff (q \iff r)$
F	F	F	T	T	T	F	T	T
F	F	T	T	T	T	T	T	F
F	T	F	T	F	T	F	T	T
F	T	T	T	T	T	T	F	F
T	F	F	T	T	T	F	F	F
T	F	T	T	T	T	F	T	T
T	T	F	F	T	T	T	T	F
T	T	T	T	T	T	T	T	T

For A For B For C, D For C, D For E For E For F For F

p	q	r	$(\neg q \vee r)$	$(q \implies r)$	$(p \implies q)$	$(\neg p \implies r)$	$(p \iff q)$	$(\neg q \iff r)$	$(\neg p \iff \neg q)$	$(q \iff r)$
F	F	F	T	T	T	F	T	F	T	T
F	F	T	T	T	T	T	T	T	T	F
F	T	F	F	F	T	F	F	T	F	F
F	T	T	T	T	T	T	F	F	F	T
T	F	F	T	T	F	T	F	F	F	T
T	F	T	T	T	F	T	F	T	F	F
T	T	F	F	F	T	T	T	T	T	F
T	T	T	T	T	T	T	F	T	T	T

Exercise 1.4. Explain, without using a truth table, why $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ is true when at least one of p, q, and r is true and at least one is false, but is false when all three variables have the same truth value.

Our answer:

The first proposition of the compound proposition is only true if there is at least one True value for p, q, or r. The second proposition is only true when there is at least one False value because all truth values are negated. Therefore if all three values are True or all three are False the compound proposition will be false.

Exercise 1.5. Show that $(p \to q) \to r$ and $p \to (q \to r)$ are not logically equivalent.

Our answer:

p	q	r	$(p \implies q)$	$(q \implies r)$	$(p \implies q) \implies r$	$p \implies (q \implies r)$
T	T	T	T	T	T	T
T	**T**	**F**	**T**	**F**	**F**	**F**
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

Exercise 1.6. How many of the disjunctions $p \vee \neg q \vee s, \neg p \vee \neg r \vee s, \neg p \vee \neg r \vee \neg s, \neg p \vee q \vee \neg s, q \vee r \vee \neg s, q \vee \neg r \vee \neg s, \neg p \vee \neg q \vee \neg s, p \vee r \vee s$, and $p \vee r \vee \neg s$ can be made simultaneously true by an assignment of truth values to p, q, r, and s?

Note: Truth values to p, q, r, and s does not mean assigning all values to be True (i.e. p can be True while q, r, and s can be False).

Our answer:

Answer: All 9 disjunctions can be made true simultaneously.

P	Q	R	S		p v ¬q v s	¬p v ¬r v s	¬p v ¬r v ¬s	¬p v q v ¬s	q v r v ¬s	q v ¬r v ¬s	¬p v ¬q v ¬s	p v r v s	p v r v ¬s
F	F	F	F		T	T	T	T	T	T	T	F	T
F	F	F	T		T	T	T	T	F	T	T	T	F
F	F	T	F		T	T	T	T	T	T	T	T	T
F	F	T	T		T	T	T	T	T	F	T	T	T
F	T	F	F		F	T	T	T	T	T	T	F	T
F	T	F	T		T	T	T	T	T	T	T	T	F
F	T	T	F		F	T	T	T	T	T	T	T	T
F	T	T	T		T	T	T	T	T	T	T	T	T
T	F	F	F		T	T	T	T	T	T	T	T	T
T	F	F	T		T	T	T	F	F	T	T	T	T
T	F	T	F		T	F	T	T	T	T	T	T	T
T	F	T	T		T	T	F	F	T	F	T	T	T
T	T	F	F		T	T	T	T	T	T	T	T	T
T	T	F	T		T	T	T	T	T	T	F	T	T
T	T	T	F		T	F	T	T	T	T	T	T	T
T	T	T	T		T	T	F	T	T	T	F	T	T