Exercise Session 1:

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Exercise 1 Let p and q be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore ", respectively

Express each of these as an English sentence.
a). $p \Rightarrow \neg q$, **b).** $\neg\, q \Rightarrow p$, **c).** $p \Leftrightarrow q$

Solution: We have two propositions

$$p := \text{"}\textit{Swimming at the New Jersey shore is allowed}\text{"}$$
$$q := \text{"}\textit{Sharks have been spotted near the shore}\text{"}$$

What is $\neg q$?

$$\neg q = \text{"}\textit{Sharks have not been spotted near the shore}\text{"}$$

a.) If swimming at the New Jersey Shore is allowed then Sharks have not been spotted near the shore.

b.) If sharks have not been spotted near the shore then swimming is allowed.

c.) Swimming is allowed iff sharks have been spotted near the shore.

Exercise 2 Determine whether each of these conditional statements is true of false.

a). If $1+1=2$ then $2+2=5$.

b). If $1+1=3$ then $2+2=4$.

c). If $1+1=3$ then $2+2=5$.

d). If monkeys can fly, then $1+1=3$

First define

$$p := 1+1=2$$
$$q := 2+2=5$$
$$r := 1+1=3$$
$$s := 2+2=4$$
$$t := "Monkeys\ can\ fly"$$

a). We can express these mathematically as $p \Rightarrow q$. Here p is True and q is False. Thus we get the truth table

p	q	$p \Rightarrow q$
T	F	F

b). We can express this mathematically as $r \Rightarrow s$. Here r is False and s is True. Thus we get the truth table

r	s	$r \Rightarrow s$
F	T	T

c). We can express this mathematically as $r \Rightarrow q$. Here r is False and q is False. Thus we get the truth table

r	q	$r \Rightarrow q$
F	F	T

d). We can express this mathematically as $t \Rightarrow r$. Here u is False (Ever seen a flying Monkey?) and q is False. Thus we get the truth table

t	r	$t \Rightarrow r$
F	F	T

Exercise 3: Construct the truth table for the following conditional statement:

$$(p \ \vee \neg q) \ \Rightarrow (p \wedge q)$$

Solution: The smallest building stones would be p and q. $\neg q$ has the truth table

p	q	$\neg q$
T	T	F
T	F	T
F	T	F
T	T	F

The truth table for $p \wedge q$ can be copied from the appendix

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth table for $p \vee \neg q$ can be constructed by lookig at the truth table of $p \ \vee q$ from the appendix

p	$\neg q$	$p \vee \neg q$
T	F	T
T	T	T
F	F	F
F	T	T

Truth table for $(p \vee \neg q) \Rightarrow p \wedge q$ can be constructed by looking at the truth table fir $p \Rightarrow q$ in the appendix

$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \Rightarrow p \wedge q$
T	T	T
T	F	F
F	F	T
T	F	F

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \Rightarrow p \wedge q$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Exercise 4: Show that $\neg(p \Rightarrow q)$ and $p \wedge \neg q$ are logically equivalent.

Solution 1: To get the truth table for $\neg(p \Rightarrow q)$ we can simply copy it from appendix.

p	q	$p \Rightarrow q$	$\neg(p \Rightarrow q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

The truth table for $p\ \wedge \neg q$ can be found by comparing to $p \wedge q$.

p	q	$\neg q$	$p \wedge \neg q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

Since they share the same truth tables they are equivalent

Solution 2:

$$\begin{aligned}
\neg(p \Rightarrow q) &= \neg(\neg p \vee q) \quad [p \Rightarrow q \ = \ \neg p \vee q],\ see\ appendix\ p.145\ of\ Course\ Script. \\
&= \neg(\neg p) \wedge \neg q\ according\ to\ the\ second\ De\ Morgan\ law\ [\neg(p \vee q) = \neg p \wedge \neg q]. \\
&= p\ \wedge \neg q\ since\ \neg(\neg p) \ = \ p.
\end{aligned}$$

Exercise 5: Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

Solution:

$$
\begin{aligned}
\neg(p \vee (\neg p \wedge q)) &= \neg p \wedge \neg(\neg p \wedge q) && \textit{De Morgan I} \\
&= \neg p \wedge (\neg(\neg p) \vee \neg q) && \textit{De Morgan I} \\
&= \neg p \wedge (p \vee \neg q) && \neg(\neg p) = p \\
&= (\neg p \wedge p) \vee (\neg p \wedge \neg q) && \textit{Distributive law I} \\
&= F \vee (\neg p \wedge \neg q) && \textit{Negation law I} \\
&= \neg p \wedge \neg q && \textit{Identity law I}
\end{aligned}
$$

Exercise 6: Determine wheter $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$ is satisfiable.

Solution: Recall that a compund proposition is **satisfiable** if there is an assignment of truth values to its vriables that makes it true.

p	q	r	$\neg q$	$(p \vee \neg q)$
T	T	T	F	T
T	F	T	T	T
F	T	T	F	F
F	F	T	T	T
T	T	F	F	T
T	F	F	T	T
F	T	F	F	F
F	F	F	T	T

Table 1: Truth table for $(p \vee \neg q)$

p	q	r	$\neg r$	$(q \vee \neg r)$
T	T	T	F	T
T	F	T	F	F
F	T	T	F	T
F	F	T	F	F
T	T	F	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Table 2: Truth table for $(q \vee \neg r)$

p	q	r	$\neg p$	$(r \vee \neg p)$
T	T	T	F	T
T	F	T	F	T
F	T	T	T	T
F	F	T	T	T
T	T	F	F	F
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T

Table 3: Truth table $(r \vee \neg p)$

p	q	r	$(p \vee \neg q)$	$(q \vee \neg r)$	$(p \vee \neg q) \wedge (q \vee \neg r)$
T	T	T	T	T	T
T	F	T	T	F	F
F	T	T	F	T	F
F	F	T	T	F	F
T	T	F	T	T	T
T	F	F	T	T	T
F	T	F	F	T	F
F	F	F	T	T	T

Table 4: Truth table for $(p \vee \neg q) \wedge (q \vee \neg r)$

p	q	r	$(p \vee \neg q) \wedge (q \vee \neg r)$	$(r \vee \neg p)$	$(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$
T	T	T	T	T	T
T	F	T	F	T	F
F	T	T	F	T	F
F	F	T	F	T	F
T	T	F	T	F	F
T	F	F	T	F	F
F	T	F	F	T	F
F	F	F	T	T	T

Table 5: Truth table for $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$

p	q	r	$((p \vee \neg q) \wedge ((q \vee \neg r) \wedge (r \vee \neg p)))$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	T	F
T	T	F	F
T	F	F	F
F	T	F	F
F	F	F	T

Hence $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$ is satisfiable.

Exercise 7: Express the statement "Some students in this class has visited" using predicates and quantifiers.

Solution: Let x denote a person. We can rephrase the statement as: There exists a person or several people x who are students in this class and who have also visited Mexico.

This statement can be divided into two statements. Let

$$M(x) := \text{"}x \textit{ has visited Mexico"}$$
$$S(x) := \text{"}x \textit{ is a student in this class.}$$

Thus, the full statements can be written as $\exists x(S(x) \wedge M(x))$

Exercise 8: Express the statement "Every student in this class has visited Mexico" using predicates and quantifiers.

Solution: Let x denote a person. We can rephrase the statement as: If you are a student in the class, it implies that you have been to either Canada or Mexico. Thus we can divide the statement into the following three statements.

$$S(x) = x \textit{ is a student in this class.}$$
$$M(x) = x \textit{ has visited Mexico.}$$
$$C(x) = x \textit{ has visited Canada.}$$

so we can write

= For every person x if x is a student in this class then x has visited Mexico or Canada
$= \forall x(S(x) \Rightarrow (C(x) \vee M(x)))$