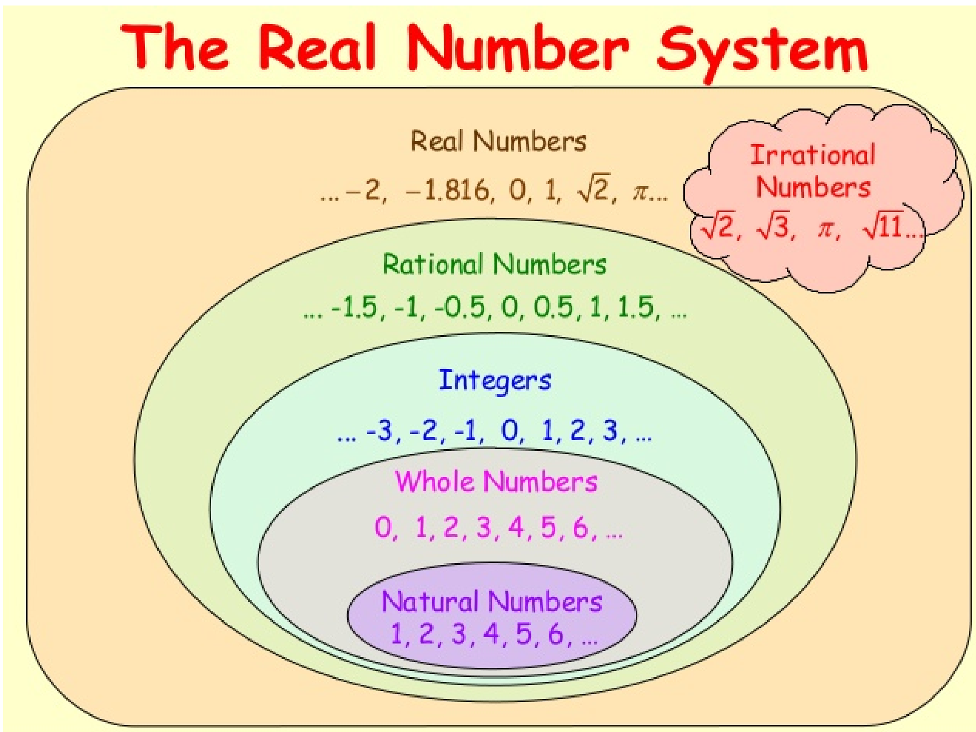
Supervision Session
Friday 2020-09-04

Some revision:

Predicate Logic:

Universe of discourse: Really important, because the output results depend on that.



$\mathbb{Z}^+$: Set of positive integers {1, 2, 3, ...}
$\mathbb{Z}^*$: Set of nonzero integers {..., -3, -2, -1, 1, 2, 3, ...}

Quantifiers:

Universal quantification: ∀ (reads to "for all", "for every", "given any"...)
Existential quantification: ∃ (reads to "there exists", "there is at least one", "for some"...)

Statement	When is True	When is False
$\forall P(x)$	P(x) is true for every x	There is an x for which P(x) is false
$\exists P(x)$	There is an x for which P(x) is true	P(x) is false for every x

Exercises:

1. Express "Some birds can sing"

Indicative answer:

By this statement we can recognize the following:
- there are some animals that are birds
- and that they can sing

We denote as: x to be an animal ($x \epsilon A$)
Let B(x) and S(x) be the statements "x is a bird" and "x can sing" respectively.

So the statement can be rephrased to: "x is a bird and x can sing", thus:

$$B(x) \ \wedge \ S(x)$$

Therefore the overall statement "some birds can sing" translates to:

$$\exists x \ (B(x) \wedge \ S(x))$$

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2. Let P (x) denote the statement "x ≤ 4." What are these truth values?
a) P (0)
b) P (4)
c) P (6)

Answer:
a) T
b) T
c) F

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3. Determine the truth value of each of these statements if the domain for all variables consists of all **integers ($\mathbb{Z}$)**.
a) $\forall n \left(n^2 \geq 0\right)$
b) $\exists n \left(n^2 \ = 2\right)$

c) $\forall n\left(n^2 \geq n\right)$
d) $\exists n\left(n^2 < 0\right)$

Answer:
a) T
b) F
c) T
d) F

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4. Translate these statements into English, where C(x) is “x is a comedian” and F (x) is “x is funny” and the domain consists of all people.
a) $\forall x(C(x) \rightarrow F(x))$
b) $\forall x(C(x) \wedge F(x))$
c) $\exists x(C(x) \rightarrow F(x))$
d) $\exists x(C(x) \wedge F(x))$

Answers:

a) Every comedian is funny
b) Every person is a funny comedian
c) There exists at least one person such that if he/she is a comedian, then he/she is funny
d) Some comedians are funny

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5. Express each of these statements using quantifiers:

a) Every Software Engineering & Management first year student must take the DIT022 course
b) Some drivers do not obey the speed limit
c) No one can keep a secret
d) There is a rabbit which is faster than all tortoises

Answers:

a)
Let F(x) be "x is a first year student"

Let C(x) be "x must take DIT022 as a course"
Universe of discourse: All SE&M students

Alternative way of the sentence: if x is a first year student, then must take the DIT022 course.

$$\forall x\big(F(x) \rightarrow C(x)\big)$$

b)
Let P(x) be "x obeys the speed limit"
Universe of discourse: All drivers

$$\exists x(\neg P(x))$$

c)
Let P(x) be "x can keep a secret"
Universe of discourse: All people

$$\neg \exists x P(x) \ \equiv \ \forall x(\neg P(x))$$

d)
We denote as x being a rabbit and y being a tortoise
Let P(x,y) be: "x is faster than y"

$$\exists x(\forall y P(x, y))$$

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6. Let P(x) be the statement "x can speak Russian" and let
Q(x) be the statement "x knows Java "
Express each ofthese sentences in terms of P(x),
Q(x), quantifiers, and logical connectives. The domain
for quantifiers consists of all students at your school.

a) There is a student at your school who can speak Russian and who knows Java.

$$\exists x(P(x) \ \wedge \ Q(x))$$

b) There is a student at your school who can speak Russian but who doesn't know Java.

$$\exists x(P(x) \ \wedge \ \neg Q(x))$$

c) Every student at your school either can speak Russian or knows Java.

$$\forall x(P(x) \ \vee \ Q(x))$$

7 . Show whether each of these conditional statements is a tautology by using truth table.

a) p → (p v q)

p	**q**	$p \vee q$	**p → (p v q)**
0	0	0	1
0	1	1	1
1	0	1	1
1	1	1	1

Since the column for p → (p v q) shows all the values to be 1, this is a tautology.

b) pV(q ∧ r)

p	**q**	**r**	**q ∧ r**	**pV(q ∧ r)**
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

This is not a tautology since the column also contains false values.

Do we know what is this called?

- Since the column has both truth and false values(or 0 and 1), this is neither a tautology nor a contradiction. It is therefore, a contingency.