Quiz 1:

Teodor Fredriksson

September 2020

Exercise 1: Consider these statements:

a.) "All lions are fierce."

b.) "Some lions do not drink coffee."

c.) "Some fierce creatures do not drink coffee."

$$\begin{aligned} P(x) &= x \text{ is a lion} \\ Q(x) &= x \text{ is fierce} \\ R(x) &= x \text{ drinks coffee} \end{aligned}$$

Assuming that the domain consists of all creatures, express the second statement (b) in the argument using quantifiers and $P(x), Q(x)$ and $R(x)$.

Solution: Let x denote "creatures" and we have that

$$\neg R(x) = x \text{ does not drink coffee}$$

then we can write statement (b) as

$$\begin{aligned} (b) &= \text{Some lions do not drink coffee} \\ &= \underbrace{\text{There exists creatures}}_{\exists x}, \underbrace{\text{these creatures are lions}}_{P(x)} \underbrace{\text{and}}_{\wedge} \underbrace{\text{these creatures do not drink coffee}}_{\neg R(x)} \\ &= \exists x (P(x) \wedge \neg R(x)) \end{aligned}$$

Exercise 2: using the same statements from Exercise 1, express the third statement (c) in the argument using quantifiers and $P(x), Q(x)$ and $R(x)$.

$$\begin{aligned} (c) &= \text{There exists creatures do not drink coffee.} \\ &= \underbrace{\text{There exists creatures}}_{\exists x}, \underbrace{\text{these creatures are fierce}}_{Q(x)} \underbrace{\text{and}}_{\wedge} \underbrace{\text{these creatures do not drink coffee}}_{\neg R(x)} \\ &= \exists x (Q(x) \wedge \neg R(x)) \end{aligned}$$

Exercise 3: Consider the compound proposition $(\neg p \Leftrightarrow \neg q) \Leftrightarrow (q \Leftrightarrow r)$. Find it's truth table.

p	q	r	$(\neg p \Leftrightarrow \neg q) \Leftrightarrow (q \Leftrightarrow r)$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	T

Table 1: Solution Exercise 3:

Exercise 4: We have the following propositions

$$\begin{aligned} p &= \text{You have the flu} \\ q &= \text{You miss that final examination} \\ r &= \text{You pass the course} \end{aligned}$$

Select the choices where there is a correct expression of each of those propositions as an English sentence.

Alternate Explanation: Determine if the expressions in the brackets represents the sentences.

1. $((p \Rightarrow \neg r) \vee (q \Rightarrow \neg r))$ If you have the flu or you miss the final examination then you do not pass the course.
2. $(p \Rightarrow q.)$ If you have the flue then you miss the final examination.
3. $(p \vee q \vee r.)$ You have the flu or you miss the final examination and you pass the course.
4. $(\neg q \Longleftrightarrow r)$. You do not miss the final examination if and only if you do not pass the course.
5. $(q \Rightarrow \neg r)$. If you miss the final examination then you pass the course.

Solution:

1.

$$\begin{aligned} &= \text{If you have the flu or you miss the final examination then you do not pass the course.} \\ &= \underbrace{\text{If you have the flu}}_{p}\ \underbrace{\text{or}}_{\vee}\ \underbrace{\text{you miss the final examination}}_{q}\ \underbrace{\text{then}}_{\Rightarrow}\ \underbrace{\text{you do not pass the course}}_{\neg r} \\ &= (p \vee q) \Rightarrow \neg r \\ &\neq (p \Rightarrow \neg r) \vee (q \Rightarrow \neg r). \end{aligned}$$

Thus the statement is false.

2.

$$
\begin{aligned}
&= \text{If you have the flue then you miss the final examination} \\
&= \underbrace{\text{If you have the flue}}_{p}\underbrace{\text{then}}_{\Rightarrow}\underbrace{\text{you miss the final examination}}_{q} \\
&= p \Rightarrow q
\end{aligned}
$$

Thus the statement is true.

3.

$$
\begin{aligned}
&= \underbrace{\text{You have the flu}}_{p}\ \underbrace{\text{or}}_{\vee}\ \underbrace{\text{you miss the final examination}}_{q}\underbrace{\text{and}}_{\wedge}\ \underbrace{\text{you pass the course.}}_{r} \\
&= p \vee q \wedge r \\
&\neq p \vee q \vee r
\end{aligned}
$$

Thus the statement is false

4.

$$
\begin{aligned}
&= \text{You do not miss the final examination if and only if you do not pass the course.} \\
&= \underbrace{\text{You do not miss the final examination}}_{\neg q}\underbrace{\text{if and only if}}_{\Leftrightarrow}\underbrace{\text{you do not pass the course}}_{\neg r} \\
&= \neg q \Longleftrightarrow \neg r \\
&\neq \neg q \Longleftrightarrow r
\end{aligned}
$$

Thus the statement is false.

5.

$$
\begin{aligned}
&= \text{If you miss the final examination then you do not pass the course.} \\
&= \underbrace{\text{If you miss the final examination}}_{q}\underbrace{\text{then}}_{\Rightarrow}\underbrace{\text{you pass the course}}_{r} \\
&= q \Rightarrow r \\
&\neq q \Rightarrow \neg r
\end{aligned}
$$

Thus the statement is false

Exercise 5: Let $Q(x)$ denote the statement "$x = x + 1$". What is the truth value of the quantification $\exists x Q(x)$?

Solution: As there is no real solution to $x = x + 1$, the statement is false.

Exercise 6: Let $C(x)$ be the statement "x has a cat", let $D(x)$ be the statement

"x has a dog" and let the statement $F(x)$ be the statement "x has a ferret". Let the domain consist of all students in your class.

Solution: Express the following statement in terms of $C(x), D(x), F(x)$, quantifiers and logical connectives: There is a student in your class which has all three animals as pets.

=There is a student in your class which has all three animals as pets.

=There is a student in your class, the student has a cat, and a dog, and a ferret.

$= \underbrace{\text{There is a student in your class}}_{\exists x} \underbrace{\text{the student has a cat}}_{C(x)} \underbrace{\text{and}}_{\wedge} \underbrace{\text{a dog}}_{D(x)} \underbrace{\text{and}}_{\wedge} \underbrace{\text{a ferret}}_{F(x)}$

$= \exists x C(x) \wedge D(x) \wedge F(x)$

Exercise 7: Determine which of these conditional statements is true.

1. If $2+2=4$, then $1+2=3$.
2. If $1+1=3$, then dogs can fly
3. If $1+1=3$, then unicorns exist.
4. If $1+1=2$, then dogs can fly.

First define

$$p := 2+2=4$$
$$q := 1+2=3$$
$$r := 1+1=3$$
$$s := 1+1=2$$
$$t := \text{"}Dogs\ can\ fly\text{"}$$
$$u := \text{"}Unicorns\ exists\text{"}$$

a). We can express these mathematically as $p \Rightarrow q$. Here p is True and q is False. Thus we get the truth table

p	q	$p \Rightarrow q$
T	F	F

b). We can express this mathematically as $r \Rightarrow t$. Here r is False and t is False. Thus we get the truth table

r	t	$r \Rightarrow t$
F	F	T

c). We can express this mathematically as $r \Rightarrow u$. Here r is False and u is False. Thus we get the truth table

r	u	$r \Rightarrow u$
F	F	T

d). We can express this mathematically as $s \Rightarrow t$. Here s is False (Ever seen a Unicorn?) and t is False. Thus we get the truth table

s	t	$s \Rightarrow t$
T	F	F

Exercise 8: Check which of the following equivalences are true.

1. $p \vee p \equiv T$
2. $p \wedge \neg p \equiv p$
3. $p \vee T \equiv T$
4. $p \wedge F \equiv F$
5. $p \vee F \equiv p$
6. $p \wedge p \equiv T$
7. $p \wedge T \equiv p$

Solution: See tables in Appendix

1. False, $p \vee p \equiv p$
2. False, $p \wedge \neg p \equiv F$
3. True
4. True
5. True
6. False $p \wedge p \equiv p$
7. True

Exercise 9: Select which statements are tautologies.

1. No

p	q	$[(\neg\, p \;\Rightarrow\; \neg\, q) \;\wedge\; (p \;\Rightarrow\; q) \;\wedge\; (\neg\, p \;\Rightarrow\; q)] \;\Rightarrow\; \neg\, p \;\wedge\; p$
T	T	F
T	F	T
F	T	T
F	F	T

2. No

p	q	$p \wedge (\neg p \Rightarrow p) \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

3. Yes

p	q	r	$\neg(p \wedge q) \wedge \neg(p \implies r) \wedge (q \Rightarrow r) \neg\, r$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

4. Yes

p	q	$p \vee (p \wedge q) \Rightarrow p$
T	T	T
T	F	T
F	T	T
F	F	T

Exercise 10: Determine which compound proposition is not satisfiable.

1. No

p	q	$(\neg p \Rightarrow q) \vee (\neg p \Rightarrow \neg q) \wedge (p \Rightarrow q) \wedge (\neg p \Rightarrow \neg q) \wedge (p \Rightarrow q))$
T	T	T
T	F	T
F	T	T
F	F	T

2. Yes

p	q	$(\neg p \Rightarrow q) \vee (p \Rightarrow \neg q) \wedge (p \Rightarrow q) \wedge (\neg p \Rightarrow \neg q)$
T	T	F
T	F	F
F	T	F
F	F	F

3. No

p	q	r	$(\neg p \vee \neg q \vee \neg r) \wedge (\neg p \vee q \vee s) \wedge (p \vee r \vee s) \wedge (\neg p \vee \neg q \vee r) \wedge (\neg p \vee r \vee s)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

4. No

p	q	$(p \Rightarrow q) \vee (\neg p \Rightarrow q) \wedge (p \Rightarrow q) \wedge (p \Rightarrow \neg q)$
T	T	T
T	F	T
F	T	T
F	F	T