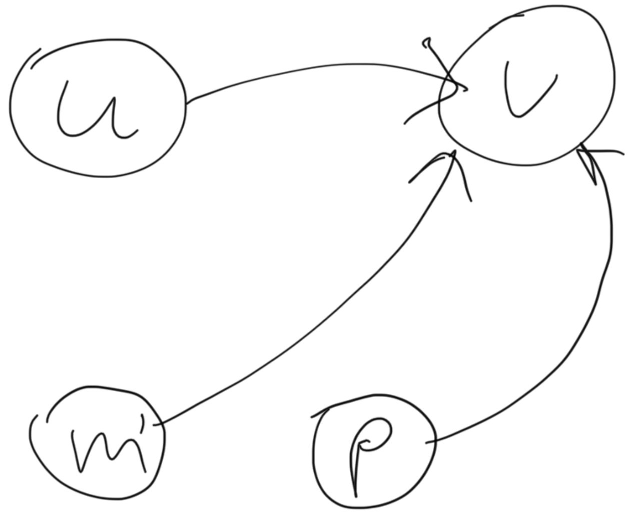
DIT-022 - Lecture 2020-09-14



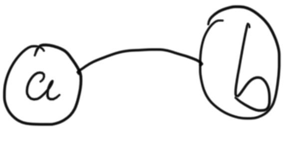
Def 19

$G = (V, E)$

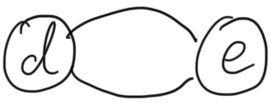
V = non-empty set of vertice

E = set of edges $V \times V$

simple graph



multigraph



loop:

Def 20: directed graph

$G = (V, E)$

V = non-empty set of verti

$E = V \times V$: ordered pair

e_j $\quad E = V \times V$, ordered pair

$e_i = (u, v)$

$e_j = (v, u)$

simple directed graph G

no loops

no multi-edges

! simple directed graph

<u>Def 21</u>: two nodes u, v are adjacent in an undirected graph when u and v are endpoints of an edge e. e is called incident with u, v.

e is connecting u, v.

<u>Def 22</u>: all neighbors of a $v \in V$ from $G = (V, E)$ is denoted as $N(v)$.

$$N(A) = \bigcup_{v \in A} N(v)$$

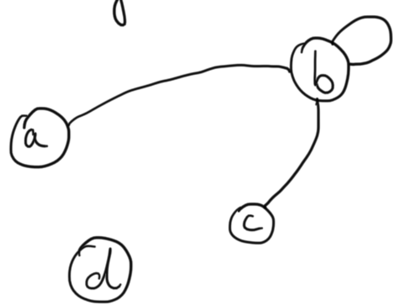
$$\underbrace{N(a) \cup N(b) \cup N(c) \ldots}_{\bigcup N(v)}$$

$$\bigcup_{v \in A} N(v)$$

Def 23: degree: number of edges incident to a node / vertex.

$\deg(v)$

loop: $+2$



$\deg(a) = 1$
$\deg(c) = 1$
$\deg(b) = 4$
$\underbrace{\deg(d) = \emptyset}_{\text{isolated.}}$

Def 24: handshaking theorem:

$G = (V, E)$

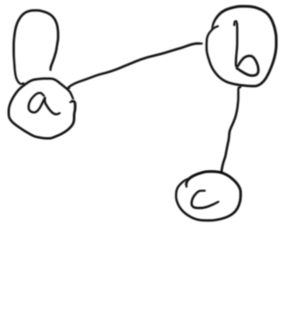
$$\underbrace{\sum_{v \in V} \deg(v)}\ = 2m$$

m = number of edg

$V = \{a, b, c, \dots, z\}$

$$\sum_{v \in V} \deg(v) = \deg(a) + \deg(b) + \deg(c) + \dots + \deg(z)$$

$m = 3$



$\deg(c) = 1$
$\deg(b) = 2$
$\deg(a) = 3$

$\sum_{v \in V} \deg(v) = 1 + 2 + \dots = 6$

$\sum_{v \in V} \deg(v) = 2\cdot$

Def 25: in-degree of edges into v:

$$\deg^-(v)$$

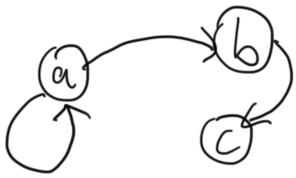
(v is the endpoint)

out-degree of edges from v:

$$\deg^+(v)$$

(v is the start point)

$$\boxed{\begin{array}{ll} \text{loop: } +1 & \text{for } \deg^+(v) \\ \phantom{\text{loop: }} +1 & \text{for } \deg^-(v) \end{array}}$$



$\deg^+(a) = 2$

$\deg^-(a) = 1$

$\deg^+(b) = 1$

$\deg^-(b) = 1$

$\deg^+(c) = 0$

$\deg^-(c) = 1$

Def 26: $G = (V, E)$ is a directed graph with m edges

$$\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = m = |E|$$

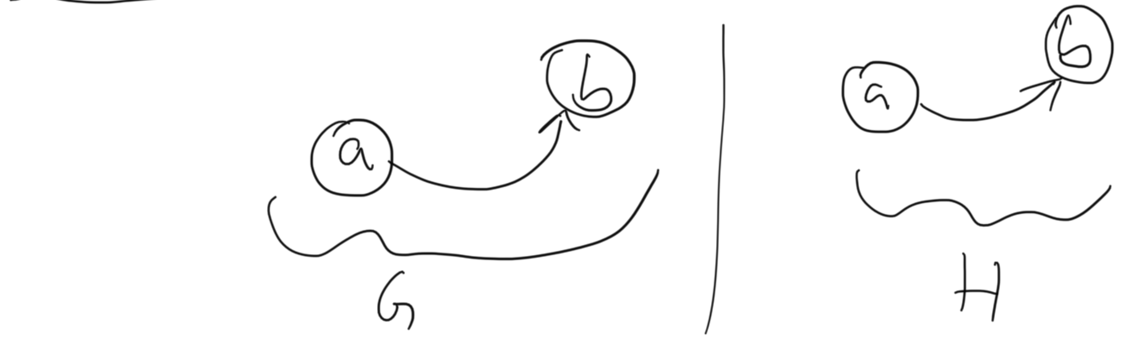
$\{ \ldots \}$

number of elem in a set

Def 27: sub-graph $H = (W, F)$ from

$G = (V, E)$

with $W \subseteq V$
and $F \subseteq E$

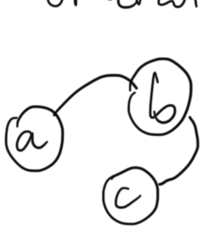


adjacency matrix

$G = (V, E)$ $\quad |V| = n$
$\quad\quad\quad\quad\quad |E| = m$

nodes: $v_1, v_2, v_3, \dots, v_n$

A is the adjacency matrix from G as $0/1$-matrix, where 1 is representing that v_i, v_j are adjacent, 0 otherwise.



$V = \{a, b, c\}$
$|V| = 3$
$E = \{(a,b), (b,c)\}$
$|E| = 2$

arriving at

leaving from

$$A = \begin{array}{c|ccc} & a & b & c \\ \hline a & 0 & 1 & 0 \\ b & 1 & 0 & 1 \\ c & 0 & 1 & 0 \end{array}$$

Δ

$$a_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{otherwise} \end{cases}$$

A depends on the chosen order; there are $n!$ different adjacency matrices ($n!$ different orders)

$n! = 1 \cdot 2 \cdot 3$

$n = 3$

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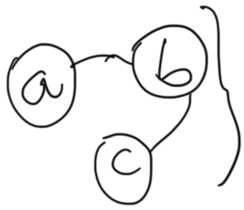
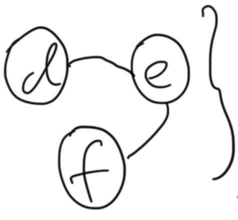
all undirected graphs have symmetric adjacency matrices

<u>Def 28</u>: Simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic, iff there exists a function f



G_1

$f: V_1 \to V_2$, where $a \in V_1$, $b \in V_1$



G_2

so that if a and b are adjacent in G_1, $f(a)$ and $f(b)$ ($f(a), f(b) \in V_2$) are adjacent in G_2 as well.

$f(a) = d$
$f(b) = e$
$f(c) = f$

in practice, there are $n!$ possible one-to-one mappings.

<u>isomorphic graphs</u>: - must have same number of nodes

- must have same number of edges

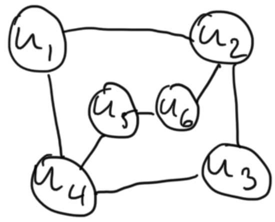
of edges

– nodes' degrees must matc
(simple graph)

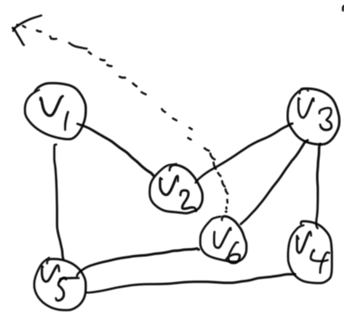
adjacency matrices:

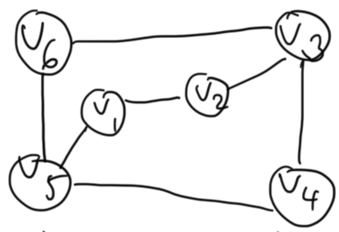
Example

G:



H:





A

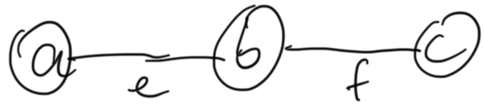
	u_1	u_2	u_3	u_4	u_5	u_6
u_1	0	1	0	1	0	0
u_2		0	1	0	0	1
u_3			0	1	0	0
u_4				0	1	0
u_5					0	1
u_6						0

	v_6	v_3	v_4	v_5	v_1	v_2
v_6	0	1	0	1	0	0
v_3		0	1	0	0	1
v_4			0	1	0	0
v_5				0	1	0
v_1					0	1
v_2						0

✓

Connectivity

"path" = sequence of edges from a starting node to an end node.



Def 29: G is undirected graph
$u, v \in V$
path p of length n $(n \in \mathbb{N}$

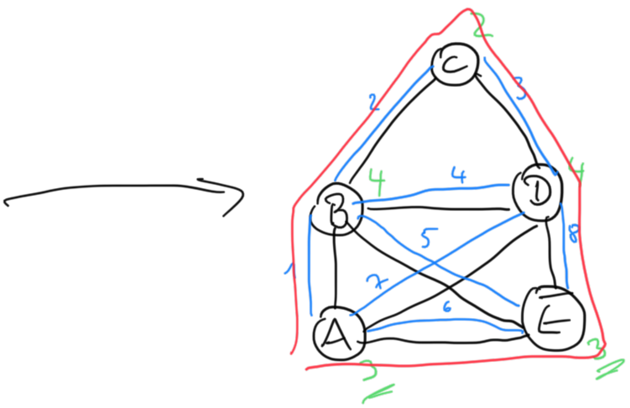
is a sequence of edges $e_1, e_2, e_3, \dots, e_n$ for which we find a sequence $x_0 = u$, $x_1 = v_1$, $x_2 = v_2, \dots, x_n = v$

so that all e_i for $i = 1, \dots, n$

the endpoints are x_{i-1} and x_i

<u>circuit</u> path that begins and ends at the same node and $n > 0$.

<u>simple circuit</u> do not use the same edge multiple times



Euler path

Ⓐ Ⓔ

<u>Def 30</u> undirected graph G is connected, when there is a path for every distinct pair of two nodes.

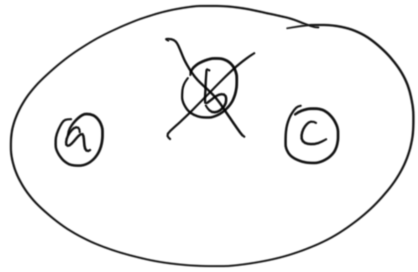
G:

a, b ✓
b, c ✗
a, c ✗

not connected.

b is cut vertex:

G



number of compon
1
2

e, f: cut edge:

1 ⟶ 2

Def 33: Euler circuit = simple circuit containing every edge from G

Euler path = simple path containing every e from G.

Def 34: connected multigraph G with $|V| \geq 2$ iff each of its nodes has an even degree

connected multigraph G with $|V| \geq 2$ if exactly 2 nodes have an odd degree

Def 35: Hamilton circuit: simple circu in G that passes every node exactly once.

Hamilton path: simple path in G tha passes every nod exactly once

sufficient conditions

Def 36: <u>Dirac's theorem</u>: if G is a simple graph with n nodes, and $n \geq 3$, such that $\deg(v)$ for every v is at least $\frac{n}{2}$, then G has a Hamilton circuit

Def 37: <u>Ore's theorem</u>: if G is a simple graph with n nodes, and $n \geq 3$ such that $\deg(u) + \deg(v) \geq$ for every pair of non-adjacent vertices $u, v \in V$, then G has a Hamilton circuit

<u>necessary condition / sufficient condition</u>

necessary condition:

Def: A condition <u>A</u> is said to be necessary for a condition <u>B</u>, if the falsity (non-occurrence) of <u>A</u> guarantees the falsity (non-occurrence) of B.

"air is necessary for human life"

sufficient condition:

Def: A condition <u>A</u> is said to be sufficient for a condition <u>B</u>, if the truth (occurrences) of <u>A</u> guarantees the truth (occurrence) of <u>B</u>.

"air is not sufficient for human life"