# Exercise Session 2:

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September 2020

**Exercise 1:** Find a phrase-structure grammar for each of these languages:

1. The set consisting of the bit strings 0, 1, and 11.

   **Solution:** We need to find a grammar $G$ such that $L(G) = \{0, 1, 11\}$, This language is finite so only need the starting values

$$
\begin{aligned}
S &\to 0 \\
S &\to 1 \\
S &\to 11
\end{aligned}
$$

2. All sets of bit strings containing only 1s.

   **Solution:** We need to find a grammar $G$ such that $L(G) = \{1, 11, ..., \underbrace{11....1}_{n}\}$.

$$
\begin{aligned}
S &\to 1 \\
S &\to 1S
\end{aligned}
$$

3. All sets of bit strings that start with 0 and end with 1.

   **Solution:** We need to find the grammar $G$ such that $L(G) = \{01, 001, 011, ..\}$

$$
\begin{aligned}
S &\to 0A \\
A &\to 0A \\
A &\to 1A \\
A &\to \lambda
\end{aligned}
$$

4. All sets of bit strings that starts with a 0 followed by an even number of 1s

   **Solution:** We need to find the grammar $G$ such that $L(G) = \{011, 01111, ..., 0\underbrace{11...1}_{2n}\}$

$$
\begin{aligned}
S &\to 0A \\
A &\to 11A \\
A &\to \lambda
\end{aligned}
$$

**Exercise 2:** Express each of these sets using a regular expression.

1. All sets consisting of strings 0, 11, and 010.

   **Solution:** A string of this set is either 0, 11 or 010 so we simply write 0|11|010.

2. All sets of bit strings of three 0s followed by two or more 0s

   **Solution:** To be clear, this is the same as saying that we need at least five 00000, to represent the uncertain amount of 0s that follow we need to end the regular expression with 0*. So in conclusion the regular expression is 000000*.

3. All sets of bit strings of odd length

   **Solution:** An odd number can be written as $2k + 1$ for integers $k$. So the first bit is represented by 0|1. This needs to be followed by uncertain amount of even strings. Thus the regular expression would be (0|1)((0|1)(0|1))∗

4. All sets of bit strings that contain exactly one 1.

   **Solution:** The string can start or end with an uncertain amount of 0s as long as there is exactly one 1, therefore the regular expression is 0*10*

5. Empty bit string and all sets of bit strings ending in 1 and not containing 000.

   **Solution:** So the string cannot end wit 0001, but 01 or 001 or you can have several 1s in a row. The regular expression would be (1|01|001)∗

**Exercise 3:** Let $V = \{S, A, B, a, b\}$ and $T = \{a, b\}$. Find the language generated by the grammar $(V, T, S, P)$ when the set of $P$ of productions consists of:

1. $S \to AB,\ A \to ab,\ B \to bb.$

   **Solution:**

   $$S \to AB \to \underbrace{ab}_{A}\underbrace{bb}_{B} = abbb$$

   Thus $L(G) = \{abbb\}$

2. $S \to AB,\ S \to aA\ A \to a,\ B \to ba$

**Solution:**

$$S \to aA \to a\underbrace{a}_{A} = aa$$
$$S \to AB \to \underbrace{a}_{A}\underbrace{ba}_{B} = aba$$

Thus $L(G) = \{aa, aba\}$

3. $S \to AB,\ S \to AA\ A \to aB,\ A \to ab,\ B \to b$

**Solution:**

$$S \to AB \to \underbrace{A}_{aB} B \to a\underbrace{B}_{b}\underbrace{B}_{b} \to abb$$
$$S \to AA \to \underbrace{A}_{aB}\underbrace{A}_{aB} \to a\underbrace{B}_{b}a\underbrace{B}_{b} = abab$$

**Exercise 4:** Explain what the productions are in a grammar if the "Backus-Naur form" for productions is as follows:

$$\begin{aligned} \text{expression} &::= (\text{expression})| \\ &\quad\ \text{expression} + \text{expression}| \\ &\quad\ \text{variable} \\ \text{variable} &::= x|y \end{aligned}$$

**Exercise 5:** Draw a diagram belonging to the table below. The starting state

Table 1:

| State | $f$ | | $g$ | |
|---|---|---|---|---|
| | **Input** | | **Input** | |
| | 0 | 1 | 0 | 1 |
| $s_0$ | $s_0$ | $s_4$ | 1 | 1 |
| $s_1$ | $s_0$ | $s_3$ | 0 | 1 |
| $s_2$ | $s_0$ | $s_2$ | 0 | 0 |
| $s_3$ | $s_1$ | $s_1$ | 1 | 1 |
| $s_4$ | $s_1$ | $s_0$ | 1 | 0 |

is state $s_0$. what is the output for "1110010101"?

**Solution:**We start at $s_0$

$$\begin{aligned}
In &= 1,\, s_0 \to s_4,\;\; Out = 1\\
In &= 1,\, s_4 \to s_0,\;\; Out = 0\\
In &= 1,\, s_0 \to s_4,\;\; Out = 1\\
In &= 0,\, s_4 \to s_1,\;\; Out = 1\\
In &= 0,\, s_1 \to s_0,\;\; Out = 0\\
In &= 1,\, s_0 \to s_4,\;\; Out = 1\\
In &= 0,\, s_4 \to s_1,\;\; Out = 1\\
In &= 1,\, s_1 \to s_3,\;\; Out = 1\\
In &= 0,\, s_3 \to s_1,\;\; Out = 1\\
In &= 1,\, s_1 \to s_3,\;\; Out = 1
\end{aligned}$$

So the output sequence is 1011011111

**Exercise 6:** Construct a deterministic finite state automation that recognizes the set of all bit strings beginning with 01

**Solution:** We need to find $M$ with $L(M) = \{01x\}$ where $x$ is any string. First we start out with and initial state $s_0$. Define $s_1$ so that $s_0 \xrightarrow{0} s_1$, then a final state $s_2$ with $s_1 \xrightarrow{1} s_2$. Finally we need a state $s_3$ with $s_3 \xrightarrow{0,1} s_3$, $s_0 \xrightarrow{1} s_3$ and $s_1 \xrightarrow{0} s_3$.

**Exercise 7:** Show that there is no finite-state automation with two states that recognizes the set of all bit strings that have one or more 1s bits and end with a 0.

**Solution:** Assume there exists such a machine with $L(M) = \{10, 110, 000110, ...\}$ and start state $s_0$ and another state $s_1$. because the empty string is noting the language but some strings are accepted, we must have $s_1$ as the only final state, with at least one transition from $s_0$ to $s_1$. Because the string 0 is not in the language, the transition from $s_0$ to $s_1$. Because the string 0 is not in the language, the transition form $s_0$ on input 0 must be to itself, so the transition from $s_0$ on the input 1 must be to $s_1$. But this contradicts the fact that 1 is not in the language.

**Exercise 8:** Construct a state-machine representing a unit-delay machine, which produces as output the input string delayed by a 0, that is, it produces as output bit string $0x_1x_2...x_{k-1}$ given the input bit string $x_1x_2...x_k$?

**Solution:**

Table 2:

| State | $f$ | | $g$ | |
|---|---|---|---|---|
| | **Input** | | **Input** | |
| | 0 | 1 | 0 | 1 |
| $s_0$ | $s_2$ | $s_1$ | 0 | 0 |
| $s_1$ | $s_1$ | $s_2$ | 1 | 1 |
| $s_2$ | $s_2$ | $s_2$ | 0 | 0 |