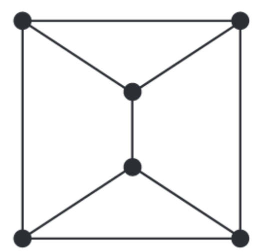
Supervision Session

Friday 2020-09-18

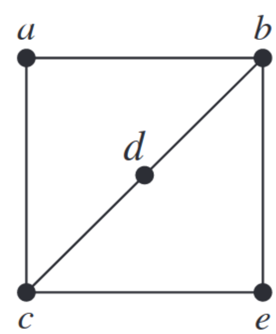
Exercise 1. Determine whether the following graphs have a Hamiton circuit.

a)



Answer:
There is a Hamilton circuit. Particularly, Dirac's theorem applies, since this graph has $n = 6$ (number of vertices), and all of them have a degree of 3, which is (at least) $\frac{n}{2}$. Furthermore, Ore's theorem also applies, because $deg(u) + deg(v) \geq n$ holds true for every pair of nonadjacent vertices u and v in the graph, since every node has a degree of 3, therefore $3+3 \geq 6$.

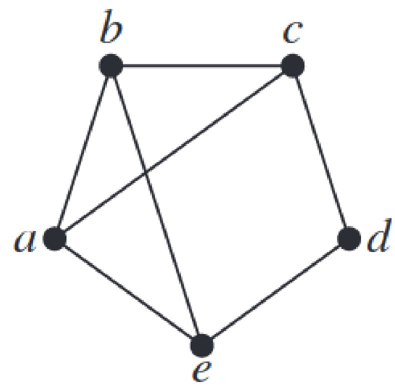
b)



Answer:
There is no Hamilton circuit in this graph.

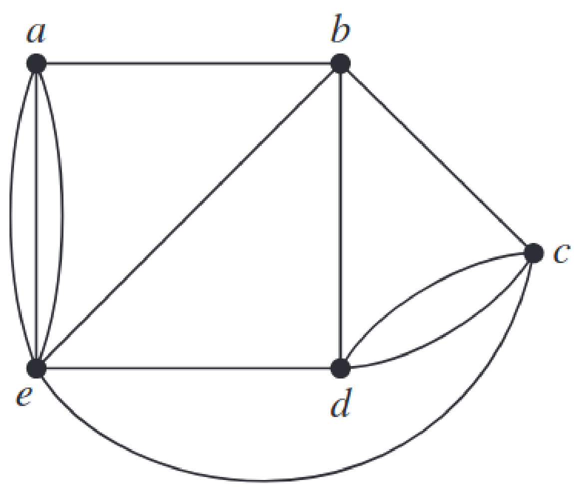
Exercise 2. Determine whether the graphs shown below have an Euler circuit. Construct an Euler circuit if one exists. If no Euler circuit exists, determine whether the graph has an Euler path. Construct an Euler path if one exists.

a)



Answer:
The graph has neither an Euler circuit nor an Euler path, due to the fact that there are four vertices of odd degree (a,b,c, and e).

b)

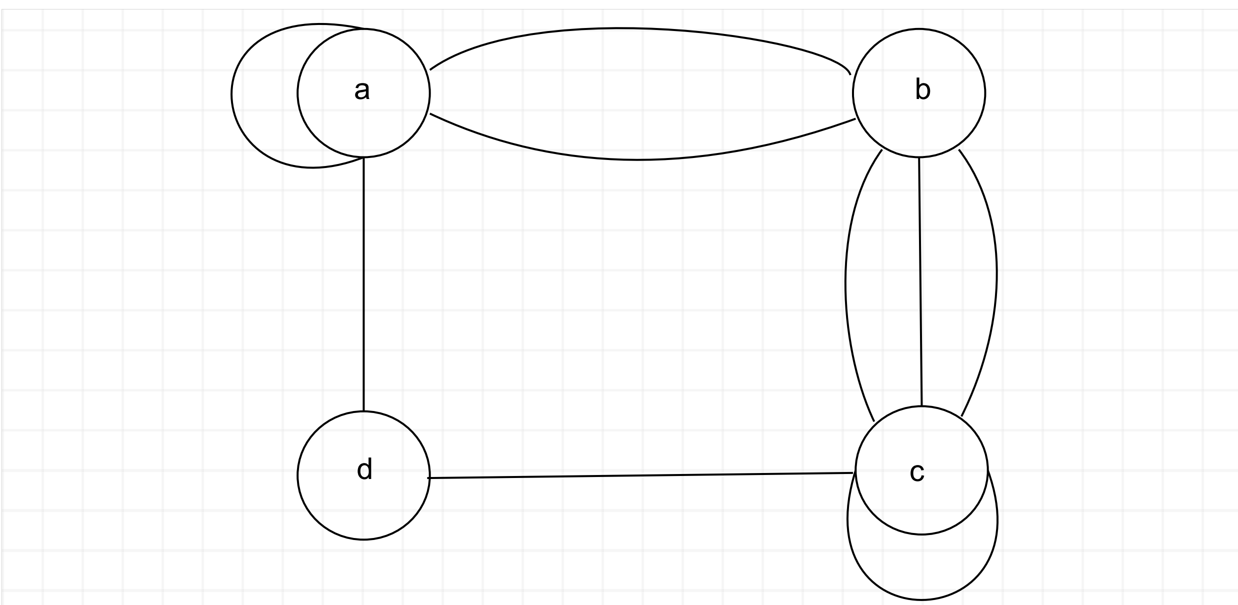


Answer:
The graph has an Euler circuit.
a, b, c, d, c, e, d, b, e, a, e, a

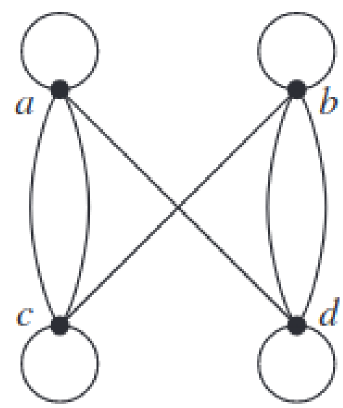
Exercise 3. Draw an undirected graph represented by the given adjacency matrix.

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Solution:



Exercise 4. Represent the given graph using an adjacency matrix.

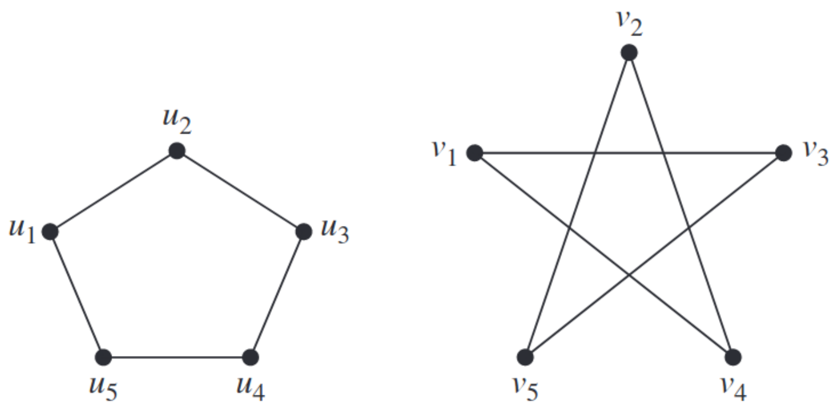


Solution:

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$

Exercise 5. Determine whether the given pairs of graphs are isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists otherwise.

a)



Answer:
They are isomorphic. One isomorphism is the following:

$$
\begin{aligned}
f(u_1) &= v_1 \\
f(u_2) &= v_3 \\
f(u_3) &= v_5 \\
f(u_4) &= v_2 \\
f(u_5) &= v_4
\end{aligned}
$$

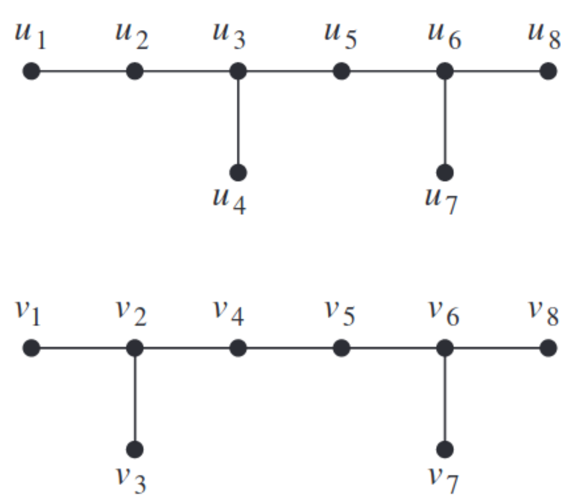
Adjacency matrix for the graph on the left:

	u_1	u_2	u_3	u_4	u_5
u_1	0	1	0	0	1
u_2	1	0	1	0	0
u_3	0	1	0	1	0
u_4	0	0	1	0	1
u_5	1	0	0	1	0

Adjacency matrix for the graph on the right:

	v_1	v_3	v_5	v_2	v_4
v_1	0	1	0	0	1
v_3	1	0	1	0	0
v_5	0	1	0	1	0
v_2	0	0	1	0	1
v_4	1	0	0	1	0

b)



Answer:
These graphs are not isomorphic. In the first graph the vertices of degree 3 are adjacent to a common vertex, which is not true for the second graph.