Exam
Logical theory part I, LOG110

2019–10–28

This exam is marked and graded anonymously using code numbers. Please enter your name and personal identity number below. Then enter only the code number on the answer sheets.

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FLoV
Göteborgs universitet

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No aids are permitted.

1. Derive the following sentences using natural deduction: (5p)
   (a) $\varphi \to \neg\bot$
   (b) $(\neg\varphi \vee \psi) \to (\varphi \to \psi)$
   (c) $\neg\forall x\varphi(x) \to \exists x\neg\varphi(x)$
2. Show that (4p)
   (a) $\not\models \exists x\varphi(x) \to \forall x\varphi(x)$
   (b) $\not\models \exists x\varphi(x) \wedge \exists x\psi(x) \to \exists x(\varphi(x) \wedge \psi(x))$
3. Show that neither $\{\neg\}$ nor $\{\wedge\}$ are (jointly) expressively adequate. (4p)
4. In propositional logic show that if $\Gamma = \{p_0, p_1, p_2, \ldots\}$ then $\Gamma \vdash \sigma$ or $\Gamma \vdash \neg\sigma$ for all formulas σ. (4p)
5. Let (4p)
   $$\mathrm{Th}(K) = \{\sigma \mid \mathfrak{M} \models \sigma \text{ for all } \mathfrak{M} \in K\}$$
   and
   $$\mathrm{Mod}(\Gamma) = \{\mathfrak{M} \mid \mathfrak{M} \models \sigma \text{ for all } \sigma \in \Gamma\}.$$
   Prove that $\Gamma \subseteq \mathrm{Th}(\mathrm{Mod}(\Gamma))$.
6. Prove that if there is no model of $T_1 \cup T_2$ then there is a sentence σ such that $T_1 \vdash \sigma$ and $T_2 \vdash \neg\sigma$. (3p)

Max points: 24. 12 points are required for Pass (G) and 18 for Pass with distinction (VG).

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