Exam
Logical theory part II, LOG110

2020–01–16

This exam is marked and graded anonymously using code numbers. Please enter your name and personal identity number below. Then enter only the code number on the answer sheets. You may write your answers in English or Swedish.

Name / Namn: . . . .

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FLoV
Göteborgs universitet

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No aids are permitted.

1. Consider the following derivation $\mathcal{D}$: (4p)

$$\dfrac{\dfrac{\dfrac{[\varphi \wedge \varphi]}{\varphi}\,\wedge\mathrm{E}}{(\varphi \wedge \varphi) \to \varphi}\,\to\mathrm{I} \qquad\qquad \dfrac{\varphi \quad \varphi}{\varphi \wedge \varphi}\,\wedge\mathrm{I}}{\varphi}\,\to\mathrm{E}$$

   (a) What is the cut rank of $\mathcal{D}$, $\mathrm{cr}(\mathcal{D})$, in terms if $r(\varphi)$?

   (b) Eliminate a maximal cut and write down the resulting derivation. What is the cut rank of that derivation?

   (c) If possible, continue eliminating cuts and write down the resulting cut free derivation.

2. Show that the following formulas are not derivable in intuitionistic logic. (4p)

   (a) $\neg\neg p \to p$.

   (b) $(p \to q \vee r) \to ((p \to q) \vee (p \to r))$

3. The halting function $f(e, n)$ is defined to be 0 if the machine M_e does not halt on input n and 1 otherwise. This function is not Turing computable. Use this fact to show that the function $g(e) = f(e, 0)$ is not Turing computable. (4p)

4. State the fixed-point lemma and prove it by using the sentence $\alpha(\ulcorner\alpha\urcorner)$, where $\alpha(x)$ is $\exists y(\theta_{\text{diag}}(x, y) \wedge \psi(y))$. (4p)

5. A theory T in the language of arithmetic is said to be ω-*complete* if whenever $T \vdash \varphi(\bar{n})$ for all $n \in \mathbb{N}$ we have $T \vdash \forall x\varphi(x)$. (4p)

   (a) By using Gödel's incompleteness theorem find a consistent theory T extending Q that is not ω-consistent and prove it. [Yes, "ω-consistent."]

   (b) Show that if T extends Q, is consistent and not ω-consistent then T is not ω-complete.

(c) Show that if T extends Q, is consistent and axiomatizable then T is not ω-complete.

6. Show that we could have defined the primitive recursive functions without using the function zero$(x) = 0$ as an initial function. I.e., show that the function zero can be defined from the other initial functions by composition and primitive recursion. (4p)

Max points: 24. 12 points are required for Pass (G) and 18 for Pass with distinction (VG).

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