

Exam

Set theory, LOG120

2019-10-30

This exam is marked and graded anonymously using code numbers. Please enter your name and personal identity number below. Then enter only the code number on each answer sheet.

Name / Namn:

Personal identity number / Personnummer:

Code number / Tentamensnummer:

No aids are permitted.

1. State the definition of an ordered pair and list all the elements of the set $\langle \{\emptyset\}, \emptyset \rangle$. (3p)
2. Show that if $A, B \neq \emptyset$ and $A \times B = B \times A$ then $A = B$. (3p)
3. Show that if $f : A \rightarrow B$, $g : B \rightarrow A$ are such that $g \circ f : A \rightarrow A$ is a bijection then f is one-one and g is onto. (4p)
4. The successor (class) function $x^+ = x \cup \{x\}$ is defined on all sets x . Show that if $x^+ = y^+$ then $x = y$. (3p)
5. (a) What is an ordinal? Define. (4p)
(b) What is a transitive set? Define.
(c) Give an example of a transitive set that is not an ordinal.
6. Show that $1 \oplus \omega = \omega$ and that $\omega \oplus 1 \neq \omega$. (4p)
7. Frege defined the natural number 1 as the set A of all sets with exactly one element, i.e., $A = \{x \mid x \approx \{\emptyset\}\}$. Show in ZF that no such set A exists. (3p)

Max points: 24. 12 points are required for Pass (G) and 18 for Pass with distinction (VG).

Fredrik Engström, fredrik.engstrom@gu.se, 031 – 786 6335