FLoV
Göteborgs universitet

EXAM 2021-01-15
Logical theory part II, LOG111

Books and lecture notes are permitted, as are computers. However, no interaction with other people (except with the teacher) during the exam is allowed.

1. (a) Show that the following formula is not derivable in intuitionistic logic: $\neg\neg p \vee \neg p$ (6p)

   (b) The relational model $\langle \{0,1,2\}, R, V \rangle$ where R is the usual ordering on $\{0,1,2\}$, $V(p_0) = \{1,2\}$, $V(p_1) = \{2\}$, and $V(p_2) = \emptyset$ shows that
   $$(p_0 \leftrightarrow p_1) \vee (p_0 \leftrightarrow p_2) \vee (p_1 \leftrightarrow p_2)$$
   is not derivable in intuitionistic logic. Show that
   $$(p_0 \leftrightarrow p_1) \vee (p_0 \leftrightarrow p_2) \vee (p_0 \leftrightarrow p_3) \vee (p_1 \leftrightarrow p_2) \vee (p_1 \leftrightarrow p_3) \vee (p_2 \leftrightarrow p_3)$$
   is also not derivable in intutitionistic logic.

   [The formulas in (b) can be interpreted as saying that out of three (or four) propositions at least two have the same truth value.]

2. Construct a Turing machine that computes the projection function $P^3_1 : \mathbb{N}^3 \to \mathbb{N}$, $P^3_1(x,y,z) = y$. [Use the definition of a Turing machine computing a function that is available in the textbook.] (4p)

3. (a) Show that the partial function defined by $f(n) = 1$ if n is even and undefined otherwise, is partial recursive. (5p)

   (b) Show that in general, given a recursive relation $R \subseteq \mathbb{N}$, the partial function defined by $f(n) = 1$ if $R(n)$ and undefined otherwise, is partial recursive.

4. A theory T in the language of arithmetic is said to be *ω-complete* if whenever $T \vdash \varphi(\bar{n})$ for all $n \in \mathbb{N}$ we have $T \vdash \forall x \varphi(x)$. (5p)

   (a) Show that if T is consistent and ω-complete then it is ω-consistent.

   (b) Show that if T extends Q, is consistent and axiomatizable then T is not ω-complete.

5. By using the fixed-point lemma let γ, φ, ψ and σ be such that (5p)
   $$Q \vdash \gamma \leftrightarrow \neg\mathsf{Prov}_Q(\ulcorner\gamma\urcorner)$$
   $$Q \vdash \varphi \leftrightarrow \exists x \mathsf{Ref}_Q(x, \ulcorner\varphi\urcorner)$$
   $$Q \vdash \psi \leftrightarrow \neg\exists x \mathsf{Ref}_Q(x, \ulcorner\psi\urcorner)$$
   $$Q \vdash \sigma \leftrightarrow (\neg\exists x \mathsf{Prf}_Q(x, \ulcorner\sigma\urcorner) \wedge \neg\exists x \mathsf{Ref}_Q(x, \ulcorner\sigma\urcorner))$$

   Which of these sentences are true in the standard model of arithmetic? Motivate your answer.

Max points: 25. 12 points are required for Pass (G) and 18 for Pass with distinction (VG).

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